

SHREWSBURY SCHOOL

MATHEMATICS PRIZE, 1956

1. A sum of money was put out at Compound Interest. The first year's interest was £65. 2s. 1d. and the fourth year's interest £73. 4s. 8d. Find the sum of money and the rate of interest per cent.

2. (i) Solve the equations :

$$\left. \begin{aligned} x^2 + xy + x &= 14 \\ y^2 + xy + y &= 28 \end{aligned} \right\}$$

(ii) Solve the equation $2x^2 + y^2 - 2xy + 4x + 4 = 0$, where x and y are real, whole numbers.

3. ABCD is a quadrilateral; BA, CD produced meet in O. Along OB, OC are drawn OP, OQ equal respectively to AB, DC. Prove that PQ is twice the join of the mid-points of AC, BD.

4. A gallon of spirit valued at £15 weighs one-quarter less than a gallon of water. Water is added to the spirit until a gallon of the mixture weighs one-fifteenth more than a gallon of the spirit. What is the value of the mixture per gallon ?

5. Show that the value of the expression

$$\frac{b^2c^2(c^2 - b^2) + c^2a^2(a^2 - c^2) + a^2b^2(b^2 - a^2)}{(b+c)(c+a)(a+b)}$$

is not altered by changing a into $a+x$, b into $b+x$, c into $c+x$.

6. Show how to construct a square which shall have two adjacent sides passing through two given points, and the intersection of the diagonals at a third given point.

7. Jones and Smith are cycling towards one another, Jones at 12 m.p.h. and Smith at 13 m.p.h. When they are 10 miles apart, a fly settles on Jones's nose, but, disliking the colour, flies straight off to Smith and settles on his. Disliking the shape of Smith's, it flies off again to Jones and, a prey to indecision, keeps flying to and fro until they meet. If it flies at 40 m.p.h., how far does it fly ?

8. AB, AC are two chords of a circle, and the tangents at B and C meet in P. The perpendicular from the centre of the circle to AB meets AC in D. Prove that PD is parallel to AB.