SHREWSBURY SCHOOL

MATHEMATICS PRIZE 1955

1. Income Tax for a married man and his wife, who both have earned incomes, is worked out separately for each. First, certain allowances are made to each of them and these are subtracted from their incomes. The remainders, which are called their taxable incomes, are taxed as follows : $2s.$ $6d.$ in the £ for the first £100, $5s.$ in the £ for the next £150, $7s.$ in the £ for the next £150, and the rest if any at $9s.$ in the £.

The allowances are as follows : the man receives (i) a Personal Allowance of £210, (ii) Earned Income Relief of two-ninths of his income, and (iii) a Children's Allowance of £85 for each child he may have under the age of 16. The wife receives (i) a Personal Allowance of £120, and (ii) Earned Income Relief of two-ninths of her income.

Make a neat table to show the tax that is due from a married man and his wife, who have three children under 16, if the man's earned income is £1350 p.a. and his wife's earned income is £270 p.a.

2. PQRS is a quadrilateral in which $P\hat{Q}R = 90°$ and $Q\hat{S}R = 45°$. Also the diagonal QS bisects $P\hat{Q}R$. Prove that $P\hat{R}Q = 2P\hat{S}Q$.

3. (i) In the equation $9\dfrac{3}{x}.y.\dfrac{5}{9} = 32$, x and y represent two different whole numbers less than 10. Find them.

(ii) Express as a fraction in its lowest terms $0\cdot17\dot{2}5\dot{5}$

4. A rectangular floor is paved with square tiles. If a diagonal is drawn, how many tiles does it cross if there are

(i) 8 rows with 12 tiles in each row,

(ii) 18 rows with 24 tiles in each row, and

(iii) m rows with n tiles in each row ?

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5.   P and Q are two points on the same side of a given straight line AB such that their perpendicular distances PL, QM from it are 1in. and 2in. respectively. LM=3in. S is a circle with centre Q and radius 1in. Construct accurately the circle which touches the circle S, passes through the point P, and has its centre on the line AB. Explain your construction and measure the radius of the circle.

6.   Find the square root of the following expression, and prove that you are correct by squaring your answer :

$$x^4 + 2x^3 + x^2 + 4y^4 - 4x^3y - 4x^2y + 8xy^3 - 4xy^2$$

7.   (i)   Find the value of $3^{\log_9 16}$

(ii)   Given that $\log 2{\cdot}45 = 0{\cdot}3892$, and $\log 2{\cdot}8 = 0{\cdot}4472$, find without tables the values of $\log 2$ and $\log 7$.

8.   (i)   Simplify :—

$$\left(\frac{a}{b}+\frac{b}{a}\right)^2+\left(\frac{b}{c}+\frac{c}{b}\right)^2+\left(\frac{c}{a}+\frac{a}{c}\right)^2-\left(\frac{a}{b}+\frac{b}{a}\right)\left(\frac{b}{c}+\frac{c}{b}\right)\left(\frac{c}{a}+\frac{a}{c}\right)$$

(ii)   Solve the equation

$$\frac{x-ab}{a+b}+\frac{x-a}{1+a}+\frac{x-b}{1+b} = a+b+1$$