

Shrewsbury School

MATHEMATICS PRIZE, 1952

1. A certain sum of money is invested at a certain rate per annum Compound Interest. The interest for the first two years is £5,125 and that for the following two years is £5,650 6s. 3d. Find the sum of money and the rate of interest.

2. Three men A, B, C who can run 854, 714, and 504 yards respectively in the same time, start together to run round a circular track, 440 yards in circumference. They agree to stop when they are all together again for the first time. How far must each man run ?

3. The Lowest Common Multiple of three numbers is 23100, and their Highest Common Factor is 5 : if two of the numbers are 700 and 770, find the possible values of the third number.

4. A stone plinth is in the form of a solid bounded by six plane faces. The base is a rectangle of sides 10-ft. and 8-ft., and the top is a rectangle of sides 9-ft. and 7-ft. parallel, respectively, to the longer and shorter sides of the base. The height of the plinth is 3-ft. ; find its volume.

5. Factorize the following :

$$(a) x^{36} - 36 \quad (b) x^4 - x^2 + 16 \quad (c) x^2 - 2bx + 6ab - 9a^2$$

6. Solve the equations :

$$(i) 3^{2x} + 9 = 10(3^x)$$

$$(ii) (a+x)^{\frac{2}{3}} + 4(a-x)^{\frac{2}{3}} = 5(a^2 - x^2)^{\frac{1}{3}}$$

7. I buy 40 animals consisting of rams at £4, pigs at £2, and oxen at £17 : if I spend £301, how many of each do I buy ?

8. Prove that four times the sum of the medians of a triangle is greater than three times and less than four times the perimeter.

9. In a triangle ABC, H is the mid-point of AB, and K is the point in AC such that $AK = 2KC$. CH, BK intersect at O. Find the ratio $\frac{CO}{OH}$.

10. ABCD is a quadrilateral inscribed in a circle ; BA, CD when produced meet at E ; O is the centre of the circle EAC ; prove that BD is perpendicular to OE.