Shrewsbury School.

MATHEMATICS PRIZE, 1949

1. Evaluate (i) $(0.3168)^{0.7523}$ (ii) $\dfrac{-.6241}{\log_{10} 0.036}$

2. Replace the stars by numbers in the following exact division :—

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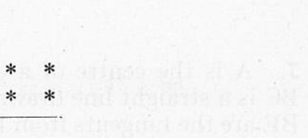
3. Last Easter a father prepared 24 Easter Eggs for his three sons. 8 were blue and contained 5/- each, 8 were red and contained 3/- each and 8 were yellow and contained 1/- each. Each son found a tricoloured array of 8 eggs awaiting him at the breakfast table and each received the same sum of money. How were the eggs distributed ?

4. If there be any number of quantities a, b, c........, prove that $(a^3 + b^3 + c^3 + \ldots\ldots\ldots) - 3(abc + abd + bcd + \ldots\ldots\ldots)$ is exactly divisible by $(a + b + c + \ldots\ldots\ldots)$, and find the quotient.

5. A man has a rectangular garden 55 yds. long by 40 yds. wide, and makes a diagonal path, 1 yd. wide, in the manner shown in the diagram. What is the area of the path ?



TURN OVER.

6. If p_5 is equal to the numerator when $a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + \cfrac{1}{a_4 + \cfrac{1}{a_5}}}}$

is expressed as a single fraction, prove that $p_5 = a_5 p_4 + p_3$, where p_4 and p_3 are equal to the numerators of similar fractions which terminate at the terms in a_4 and a_3 respectively.

By dividing each side of this result by p_4, prove that

$$\frac{p_5}{p_4} = a_5 + \cfrac{1}{a_4 + \cfrac{1}{a_3 + \cfrac{1}{a_2 + \cfrac{1}{a_1}}}}$$

7. A is the centre of a circle and B is any point outside it. BC is a straight line drawn from B to cut the circle at C. BD, BE are the tangents from B to the circle. Through C a straight line is drawn perpendicular to BC to cut AD in T and AE in U.

Prove that $A\hat{T}B = A\hat{B}U$.

8. A and B are fixed points on the circumference of a circle, centre O. X is a variable point on the minor arc AB, and Y and Z are the feet of the perpendiculars from X onto OA and OB. Prove that YZ is of constant length.