SHREWSBURY SCHOOL

MATHEMATICS PRIZE, 1948

1. A loudspeaker coil is wound on a cylindrical cardboard former of diameter $\frac{1}{2}$ inch and of length $\frac{1}{4}$ inch. The outer diameter of the coil is 1 inch, and the diameter of the wire 1/100 inch. The wire is started at one end of the cylinder and is turned around it until the curved surface is entirely covered; the windings are then continued back over the first layer, and so on. Assuming, for the purpose of calculation, that each turn is exactly circular and that each layer lies wholly outside the layer below, find the length of the wire.

2. The population of England and Wales in 1801 was 8,893,000 and in 1901 was 32,528,000. Assuming the population to increase annually according to the compound interest law, estimate the year in which the population would reach 40,000,000.

3. Replace the crosses by numbers in the following exact division:

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x4x)9xxx5(xxx
     xxx
     2xxx
     xxxx
       2xx
       xxx
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4. Find the values of x which satisfy the equations

(i) $\sqrt{\frac{x}{a}} + \sqrt{\frac{a}{x}} = \sqrt{\frac{x}{b}} + \sqrt{\frac{b}{x}}$, where a and b are both positive.

(ii) $7^{3x+2} + 4^{x+2} = 7^{3x+1} + 4^{x+3}$

5. Simplify $\frac{1}{(a+b)(b+c)}\left\{\frac{(a+b)^3 - (b+c)^3}{a-c} - \frac{(a+b)^3 + (b+c)^3}{a+2b+c}\right\}$

6. If f(x) denote the sum of a number of terms containing integral powers of x, and be divided by $(x-\alpha)$, show that the remainder is $f(\alpha)$. Also, if f(x) be divided by $(x-\alpha)(x-\beta)$, show that the remainder is px+q, where $p = \frac{f(\alpha) - f(\beta)}{\alpha - \beta}$ and $q = \frac{\beta f(\alpha) - \alpha f(\beta)}{\beta - \alpha}$

7. Show that the product of the Lowest Common Multiple and the Highest Common Factor of any two numbers is equal to the product of the numbers.

H denotes the Highest Common Factor, and L the Lowest Common Multiple, of three numbers. H_1, H_2, H_3 are the Highest Common Factors of the same numbers taken in pairs; L_1, L_2, L_3 are the corresponding Lowest Common Multiples. Show that

$$\frac{L_1 L_2 L_3}{H_1 H_2 H_3} = \frac{L^2}{H^2}$$

8. Draw a triangle of sides $2\frac{1}{2}$, 3 and 4 inches respectively, and construct geometrically the line, parallel to the 3 inch side, which bisects the area of the triangle. (A solution involving calculation, or measurement of any lengths other than of the sides of the original triangle, will not be accepted.)