Trying To Solve

$$\sum_{k=0}^{n} k^x = 0$$

Suppose $n = 2$ in which case the equation becomes

$$1^x + 2^x + 3^x = 0$$

This certainly can't have any real valued solutions since, as $1^x = 1$ for all real x, gives

$$1 + 2^x + 3^x = 0$$

$$2^x + 3^x = -1$$

And $a^x > 0$ for all real x and all real $a > 0$

So the search for a solution needs the extended number system of complex numbers.

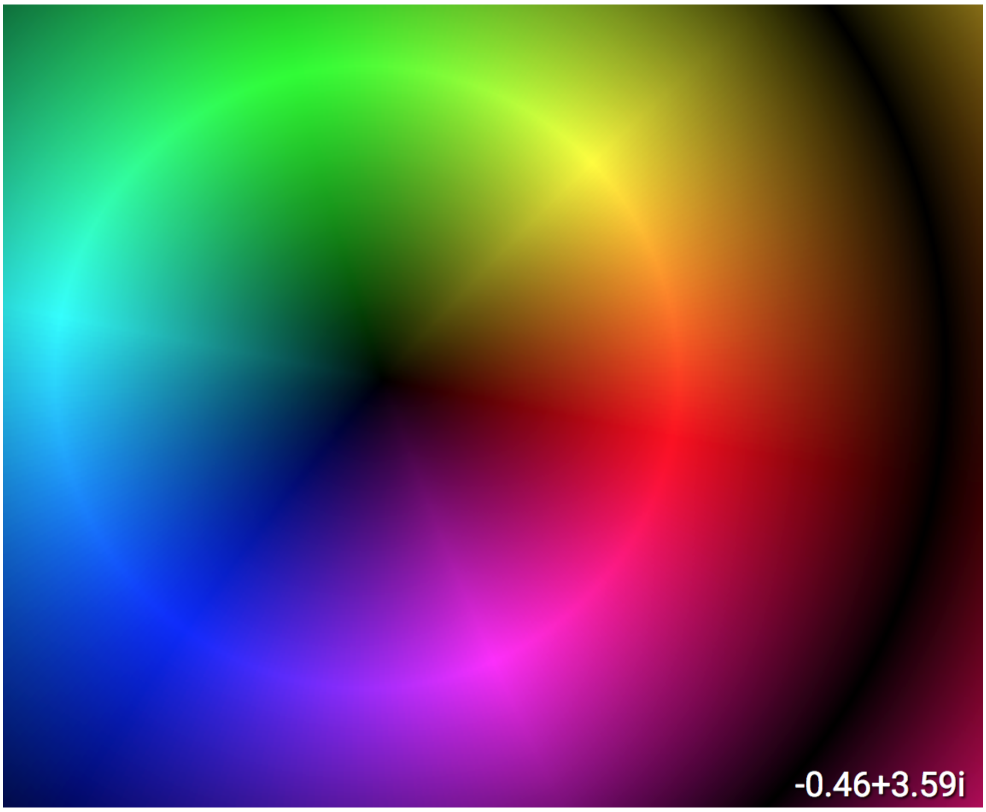
If Wolfram Alpha is asked to solve,

$$2^x + 3^x = -1$$

it will give numerical solutions

$$x = -0.454397 \pm 3.59817i$$

The zero at -0.45+3.60 can be looked at on a complex function graph plotter,



where it is seen at the centre of the coloured circle.

To find this zero, Newton Raphson iteration could be used, which works just as well with complex numbers as it does with the reals.
Starting with x_0 = -1 + πi the iterates are;

$$x_0 = -1 + \pi i$$

$$x_1 = -0.501820 + 3.86512i$$

$$x_2 = -0.487049 + 3.58176i$$

$$x_3 = -0.454116 + 3.59875i$$

$$x_4 = -0.454397 + 3.59817i$$

Although Wolfram Alpha found two solutions there may be others.
In fact. for the complex zeros

$$|2^z + 3^z| < 1 \quad for \quad Re(z) > 1$$

$$and \quad \left| 3^{-z} + \left(\frac{3}{2}\right)^{-z} \right| < 1 \quad for \quad Re(z) < -1$$

so the zeros are located in a strip Re(x) $\in$ [-1,1] and by almost periodicity there are infinitely many of them, almost linearly spaced.

Below are some results that may help explore the landscape further;

$$\sum_{k=0}^{n} z^n = 0$$

$n = 3 : -0.454397 + 3.59817i$
$n = 4 : -0.625971 + 3.12712i$
$n = 5 : -0.714285 + 2.83349i$
$n = 6 : -0.767633 + 2.62901i$
$n = 7 : -0.803209 + 2.47644i$
$n = 8 : -0.828584 + 2.35711i$
$n = 9 : -0.847585 + 2.26049i$
$n = 10 : -0.862348 + 2.18022i$
$n = 11 : -0.874153 + 2.11212i$
$n = 12 : -0.883812 + 2.05341i$
$n = 13 : -0.891868 + 2.00209i$
$n = 14 : -0.898692 + 1.95672i$
$n = 15 : -0.904551 + 1.91623i$
$n = 16 : -0.909640 + 1.87978i$
$n = 17 : -0.914103 + 1.84675i$
$n = 18 : -0.918051 + 1.81661i$
$n = 19 : -0.921571 + 1.78897i$
$n = 20 : -0.924730 + 1.76349i$

Resources : The Complex Grapher
(Free interactive Complex Function Graph Plotter)
https://www.CompexGrapher.com
Wolfram Alpha
(Free Symbolic Manipulator)
https://www.WolframAlpha.com

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