

## Lesson 5

### Further A-Level Pure Mathematics, Core 2 Complex Numbers II

#### 5.1 Trigonometric Identities, Type 1

Thanks to Euler's Relation and de Moivre's theorem, the world of complex numbers is a natural setting in which to generate trigonometric identities.

One fruitful approach is to consider an expression of the form  $(\cos \theta + i \sin \theta)^n$  for some integer value of  $n$ . By then expanding the brackets in two different ways, and setting the resulting expressions equal to each other, many useful trigonometric identities are obtained.

#### 5.2 Example

By expanding the brackets of  $(\cos \theta + i \sin \theta)^2$  in two different ways, derive two well known trigonometric identities.

Teaching Video : <http://NumberWonder.co.uk/v9099/5.mp4>



[ 6 marks ]

### 5.3 Exercise

*Any solution based entirely on graphical  
or numerical methods is not acceptable*

Marks Available : 50

#### Question 1

By expanding the brackets of  $(\cos \theta + i \sin \theta)^3$  in two different ways, derive the following two well known trigonometric identities, frequently obtained “the long way” in the single A-Level examination papers,

$$\cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta \quad \text{and} \quad \sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta$$

[ 6 marks ]

**Question 2**

By means of the substitution  $x = \cos \theta$ , and making use of the trigonometric identity  $\cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta$ , to find all three solutions to the equation,

$$8x^3 - 6x - 1 = 0$$

Give your answers to 3 decimal places

[ 5 marks ]

**Question 3**

This question is about finding all the solution to the equation

$$\cos 5\theta + 5 \cos 3\theta = 0 \quad 0 \leq \theta < \pi$$

You are given that  $\cos 5\theta = 16 \cos^5 \theta - 20 \cos^3 \theta + 5 \cos \theta$

and also that  $\cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta$

Give your answers to 3 decimal places

[ 6 marks ]

**Question 4**

*Further A-Level Examination Question from June 2014, FP2(R), Q7 (Edexcel)*

(i) Use de Moivre's theorem to show that,

$$\sin 5\theta = 16 \sin^5 \theta - 20 \sin^3 \theta + 5 \sin \theta$$

[ 5 marks ]

(b) Hence find the five distinct solutions of the equation

$$16x^5 - 20x^3 + 5x + \frac{1}{2} = 0$$

giving your answers to 3 decimal places where necessary.

[ 5 marks ]

(c) Use the identity given in (a) to find

$$\int_0^{\frac{\pi}{4}} (4 \sin^5 \theta - 5 \sin^3 \theta) d\theta$$

expressing your answer in the form  $a\sqrt{2} + b$  where  $a$  and  $b$  are rational numbers.

[ 4 marks ]

**Question 5**

(i) Use de Moivre's theorem to show that

$$\sin 4\theta = 4 \cos^3 \theta \sin \theta - 4 \cos \theta \sin^3 \theta$$

[ 4 marks ]

(ii) Hence, or otherwise, show that,

$$\tan 4\theta = \frac{4 \tan \theta - 4 \tan^3 \theta}{1 - 6 \tan^2 \theta + \tan^4 \theta}$$

[ 4 marks ]

- ( iii ) Use your answer to part (b) to find, to 2 decimal places, the four solutions of the equation,

$$x^4 + 4x^3 - 6x^2 - 4x + 1 = 0$$

[ 5 marks ]

**Question 6**

*Further A-Level Examination Question from June 2018, FP2, Q7(a) (Edexcel)*

Use de Moivre's theorem to show that

$$\cos 7\theta = 64 \cos^7 \theta - 112 \cos^5 \theta + 56 \cos^3 \theta - 7 \cos \theta$$

**[ 6 marks ]**

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Teachers may obtain detailed worked solutions to the exercises by email from [mhh@shrewsbury.org.uk](mailto:mhh@shrewsbury.org.uk)