Lesson 10

Further A-Level Pure Mathematics, Core 2

Complex Numbers II

10.1 Revision

Any solution based entirely on graphical or numerical methods is not acceptable

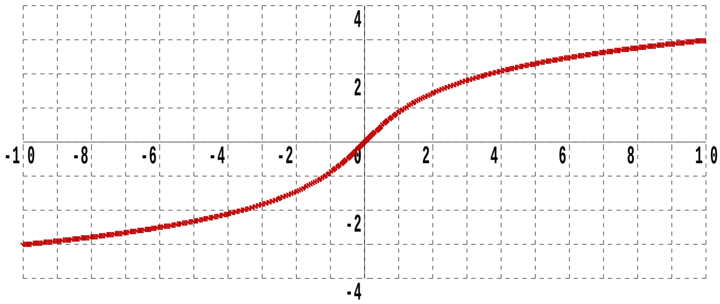
Marks Available : 40

Question 1

(i) Simplify $\left(\sqrt{x^2 + 1} - x\right)\left(\sqrt{x^2 + 1} + x\right)$

[1 mark]

(ii) The graph is of the function $f(x) = ln\left(x + \sqrt{x^2 + 1}\right)$



Given that the graph would seem to have half turn rotational symmetry about the origin does this suggest that the function is even or odd ?

[1 mark]

(iii) Prove that the function is either an even function or an odd function, as dictated by your answer to part (ii)

[4 marks]

Question 2

Show that $\dfrac{cos\, 5\theta \;+\; \mathrm{i}\, sin\, 5\theta}{cos\, 2\theta \;+\; \mathrm{i}\, sin\, 2\theta}$ can be expressed in the form $cos\, n\theta \;+\; \mathrm{i}\, sin\, n\theta$,

where n is an integer to be found

[4 marks]

Question 3

Prove that $\dfrac{e^{\mathrm{i}\theta} \;-\; 1}{e^{\mathrm{i}\theta} \;+\; 1} \;=\; \mathrm{i}\, tan\left(\dfrac{x}{2}\right)$

[5 marks]

Question 4

Further A-Level Examination Question from June 2018, FP2, Q2(a) (MEI)

(i) Use de Moivre's theorem to prove that

$$cot\,4\theta = \frac{1 - 6\,tan^2\,\theta + tan^4\,\theta}{4\,tan\,\theta\left(1 - tan^2\,\theta\right)}$$

[5 marks]

(ii) Hence express the roots of the equation

$$x^4 + 4x^3 - 6x^2 - 4x + 1 = 0$$

in exact trigonometric form

[4 marks]

Question 5

Further A-Level Examination Question from June 2009, FP2, Q3(b) (MEI)

The infinite series C and S are defined as follows,

$$C = \cos\theta + \frac{1}{3}\cos 3\theta + \frac{1}{9}\cos 5\theta + \ldots$$

$$S = \sin\theta + \frac{1}{3}\sin 3\theta + \frac{1}{9}\sin 5\theta + \ldots$$

By considering $C + \mathrm{i}S$, show that $C = \dfrac{3\cos\theta}{5 - 3\cos 2\theta}$

and find a similar expression for S

[11 marks]

Question 6

Given that $z = e^{\frac{\pi}{n}\mathrm{i}}$ where n is a positive integer, prove that

$$1 + z + z^2 + \ldots + z^n = \mathrm{i}\,cot\left(\frac{\pi}{2n}\right)$$

[5 marks]