

## Lesson 4

### Further A-Level Pure Mathematics Matrix Systems of Equations : Core 1

#### 4.1 Three Equations, Three Unknowns

The previous two lessons have developed the mathematics necessary to solve a set of three simultaneous equations in three unknowns using matrix methods.

The strategy employed is exactly the same as that used in Lesson 1 when questions of two equations in two unknowns were tackled.

#### Example

Use your calculator to assist in finding the unique solution to the system of equations,

$$2x + 4y - z = 12$$

$$x - y + 4z = 6$$

$$4x + 5y - z = 17$$

#### Teaching Instructions :

How to answer this question using a CASIO fx-991EX is presented on the next page

[ 4 marks ]

### Calculator Assisted Solution using the CASIO CLASSWIZ fx-991EX

First write the system of equations as a matrix equation,

$$\begin{pmatrix} 2 & 4 & -1 \\ 1 & -1 & 4 \\ 4 & 5 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 12 \\ 6 \\ 17 \end{pmatrix}$$

In what follows  $\mathbf{A} = \begin{pmatrix} 2 & 4 & -1 \\ 1 & -1 & 4 \\ 4 & 5 & -1 \end{pmatrix}$  and  $\mathbf{B} = \begin{pmatrix} 12 \\ 6 \\ 17 \end{pmatrix}$

Use the calculator to get the inverse of matrix  $\mathbf{A}$  as follows,

- Turn the calculator **ON** and **MENU 4** to get into matrix mode
- Press **1** to define matrix  $\mathbf{A}$
- Press **3** and **3** again to specify 3 rows and 3 columns for matrix  $\mathbf{A}$
- Enter the nine elements of the matrix  $\mathbf{A}$  pressing = after each entry
- Press **AC** to tell the calculator the matrix  $\mathbf{A}$  is now defined
- Press **OPTN 3** to initiate a calculation involving matrix  $\mathbf{A}$
- Press the button  $x^{-1}$  followed by =
- Scroll through the elements of the inverse matrix and write down,

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{21} \begin{pmatrix} -19 & -1 & 15 \\ 17 & 2 & -9 \\ 9 & 6 & -6 \end{pmatrix} \begin{pmatrix} 12 \\ 6 \\ 17 \end{pmatrix}$$

The above line of working is worth half marks.

Now the calculator will be used to perform the matrix multiplication  $\mathbf{A}^{-1}\mathbf{B}$  and so yield the values of  $x$ ,  $y$  and  $z$

- Press **MENU 4** to again enter the Define Matrix screen
- Press **2** to define matrix  $\mathbf{B}$
- Press **3** and **1** to specify 3 rows and 1 column for matrix  $\mathbf{B}$
- Enter the three elements of the matrix  $\mathbf{B}$  pressing = after each entry
- Press **AC** to tell the calculator the matrix  $\mathbf{B}$  is now defined
- Press **OPTN 3** to initiate a calculation involving matrix  $\mathbf{A}$
- Press the button  $x^{-1}$  followed by  $\times$
- Press **OPTN 4** to enter matrix  $\mathbf{B}$  into the evolving calculation
- Now press = and write down,

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$$

$\therefore$  The unique solution is  $x = 1$ ,  $y = 3$ ,  $z = 2$

[ 4 marks ]

## 4.2 Exercise

*Any solution based entirely on graphical  
or numerical methods is not acceptable*

Marks Available : 30

### Question 1

*Further A-Level Examination Question from May 2018, Core 1, Q1 (a), (b)*

$$\mathbf{M} = \begin{pmatrix} 2 & 1 & -3 \\ 4 & -2 & 1 \\ 3 & 5 & -2 \end{pmatrix}$$

- (a) Find  $\mathbf{M}^{-1}$  giving each element in exact form.

[ 2 marks ]

- (b) Solve the simultaneous equations,

$$2x + y - 3z = -4$$

$$4x - 2y + z = 9$$

$$3x + 5y - 2z = 5$$

[ 2 marks ]

**Question 2**

In the following system of equations,  $a$  is an unknown constant,  $a \neq -2$ ,

$$x - y + z = 4$$

$$4x + z = 2a$$

$$2x + ay + 2z = a$$

- (i) Construct a suitable matrix equation with a view to preparing to solve this system of equations by matrix methods.

[ 1 mark ]

- (ii) Find, in terms of  $a$ , an expression for the determinant of the matrix,

$$\mathbf{S} = \begin{pmatrix} 1 & -1 & 1 \\ 4 & 0 & 1 \\ 2 & a & 2 \end{pmatrix}$$

[ 2 marks ]

- (iii) From  $\mathbf{S}$ , form the matrix of minors,  $\mathbf{M}$ , in terms of  $a$

[ 4 marks ]

( iv ) From  $\mathbf{M}$ , form the matrix of cofactors,  $\mathbf{C}$ , in terms of  $a$

[ 1 mark ]

( v ) Write down the transpose,  $\mathbf{C}^T$ , of the matrix of cofactors, in terms of  $a$

[ 1 mark ]

( vi ) Write down in terms of  $a$  the inverse matrix  $\mathbf{S}^{-1}$

[ 1 mark ]

( vii ) Find, in terms of  $a$ , the values of  $x$ ,  $y$ , and  $z$

[ 2 marks ]

( viii ) Show that if  $a = 3$ , the values of  $x$ ,  $y$  and  $z$  are integers.

[ 2 marks ]

**Question 3**

Three planes  $A$ ,  $B$  and  $C$  are defined by the following equations;

$$A : x + y + z = 3$$

$$B : 2x - y - 2z = 0$$

$$C : 3x - 2y + z = -1$$

By constructing and solving a suitable matrix equation, show that these three planes intersect at a single point and find the coordinates of that point

[ 5 marks ]

**Question 4**

Use your calculator to find the inverse of,

$$\begin{pmatrix} 1 & 1 & 0 & 1 \\ 2 & 3 & 1 & 4 \\ 0 & 1 & 2 & 2 \\ 0 & 2 & 3 & 5 \end{pmatrix}$$

[ 4 marks ]

**Question 5**

Find the inverse of the matrix  $\begin{pmatrix} 1 & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & 1 \end{pmatrix}$  where  $k$  is a constant

[ 3 marks ]