Lesson 2

2.1 Intersecting Lines In Three Dimensions

In two dimensions if two distinct lines are not parallel they must have a point of intersection. In three dimensions the same is not true; it is possible for two lines that are not parallel lines to not intersect. Such lines are said to be **SKEW**.

2.2 Example

Determine if the following lines intersect or if they are skew.

$$\boldsymbol{r}_1 = \begin{pmatrix}1\\2\\3\end{pmatrix} + \lambda\begin{pmatrix}3\\2\\-1\end{pmatrix} \quad \text{and} \quad \boldsymbol{r}_2 = \begin{pmatrix}9\\2\\5\end{pmatrix} + \mu\begin{pmatrix}-1\\-2\\1\end{pmatrix}$$

[4 marks]

2.3 Exercise

Any solution based entirely on graphical
or numerical methods is not acceptable
Marks Available : 32

Question 1

(i) Show that the following lines intersect;

$$\boldsymbol{r}_1 = \begin{pmatrix} 2 \\ 3 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 4 \\ 1 \end{pmatrix} \quad \text{and} \quad \boldsymbol{r}_2 = \begin{pmatrix} -6 \\ -3 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ 1 \\ -2 \end{pmatrix}$$

[4 marks]

(ii) Find the coordinates of the point of intersection.

[1 mark]

Recall that in three dimensions the scalar product is;

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \bullet \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

$$a_1 b_1 + a_2 b_2 + a_3 b_3 = \sqrt{a_1{}^2 + a_2{}^2 + a_3{}^2}\ \sqrt{b_1{}^2 + b_2{}^2 + b_3{}^2}\ \cos\theta$$

(iii) Use this to find, to the nearest 0.1°, the acute angle between the lines. (Remember to use the direction part of the lines !)

[3 marks]

Question 2

C4 Examination Question from June 2007, Q5

The line l_1 has equation;

$$\boldsymbol{r} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

The line l_2 has equation;

$$\boldsymbol{r} = \begin{pmatrix} 1 \\ 3 \\ 6 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$$

(a) Show that l_1 and l_2 do not meet.

[4 marks]

The point A is on l_1 where $\lambda = 1$, and the point B is on l_2 where $\mu = 2$

(b) Find the cosine of the acute angle between AB and l_1

[6 marks]

Question 3

C4 Examination Question from June 2009, Q7

Relative to a fixed origin O, the point A has position vector $8\,\boldsymbol{i} + 13\,\boldsymbol{j} - 2\,\boldsymbol{k}$,
the point B has position vector $10\,\boldsymbol{i} + 14\,\boldsymbol{j} - 4\,\boldsymbol{k}$
and the point C has position vector $9\,\boldsymbol{i} + 9\,\boldsymbol{j} + 6\,\boldsymbol{k}$

The line l passes through the points A and B

(a) Find a vector equation for the line l

[3 marks]

(b) Find $\left|\overrightarrow{CB}\right|$

[2 marks]

(c) Find the size of the acute angle between the line segment CB and the line l, giving your answer in degrees to 1 decimal place.

[3 marks]

(d) Find the shortest distance from the point C to the line l

[3 marks]

The point X lies on l
Given that the vector $\overrightarrow{CX}$ is perpendicular to l

(e) find the area of triangle CXB, giving your answer to 3 significant figures.

[3 marks]

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