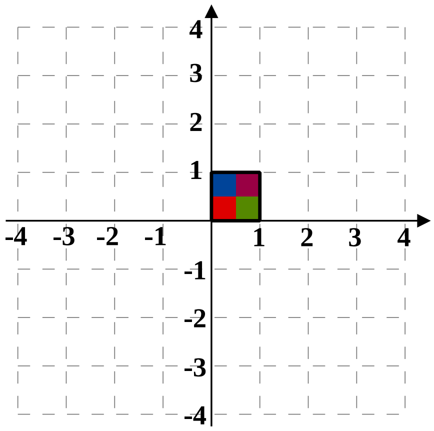
4.1 The Unit Square

Faced with an unfamiliar 2×2 transformation matrix, one way to investigate its properties is to apply it to a unit square.
Written as a multipoint matrix the unit square to use is,

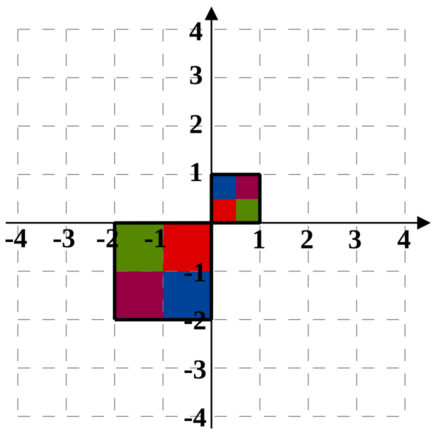
$$\mathbf{U} = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

which is visualised as,



Consider the matrix, $\mathbf{M} = \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$ used in Exercise 3.2, Question 1.

MU	$\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$
$\begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$	$\begin{pmatrix} 0 & -2 & -2 & 0 \\ 0 & 0 & -2 & -2 \end{pmatrix}$



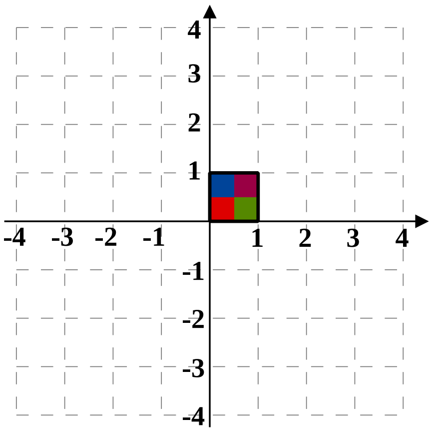
The transformation can be seen to be a rotation of 180° about the origin and an enlargement of Length Scale Factor 2.
The Area Scale Factor is 4, which is also given by *det* $\mathbf{M}$ = + 4.
The + sign indicates that the orientation is unchanged. In both the original shape and the image the colours go red, green, purple, blue in anticlockwise order.

4.2 Exercise

Question 1

A matrix, **N**, is used as a transformation, where **N** $= \begin{pmatrix} -2 & -2 \\ -2 & 2 \end{pmatrix}$

(i) Apply **N** to the unit square and plot the resulting shape on the graph below.



(ii) Calculate the determinant of **N**

(iii) Explain carefully what the sign of the determinant tells you about the transformation, **N**

(iv) What is the Length Scale Factor of the transformation ?

(v) What is the Area Scale Factor of the transformation ?

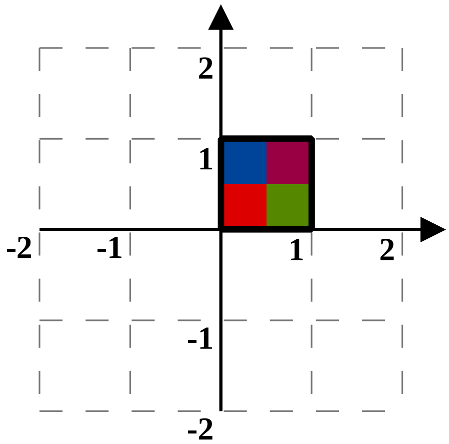
Question 2
Complete the following,
NOTE : the first entry is the identity matrix, **I**, which leaves points as they are.

A Catalogue of Two-Dimensional Transformations

$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

	$\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$
$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & & & \\ 0 & & & \end{pmatrix}$

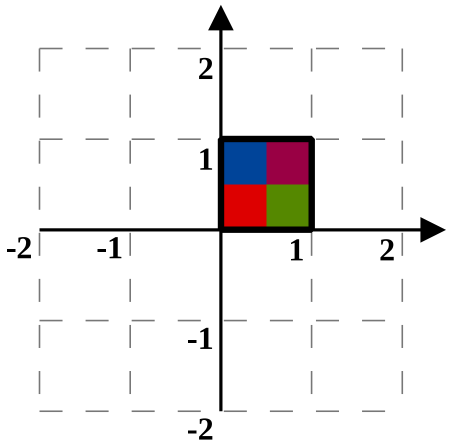
Description :



$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

	$\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$
$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & & & \\ 0 & & & \end{pmatrix}$

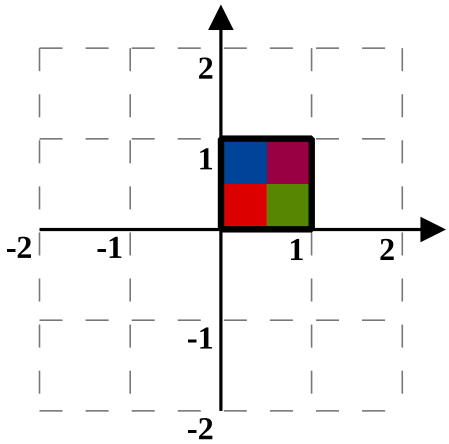
Description :



$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

	$\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$
$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 0 & & & \\ 0 & & & \end{pmatrix}$

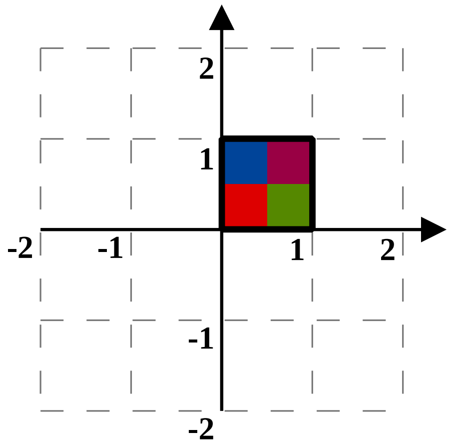
Description :



$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

	$\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$
$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & & & \\ 0 & & & \end{pmatrix}$

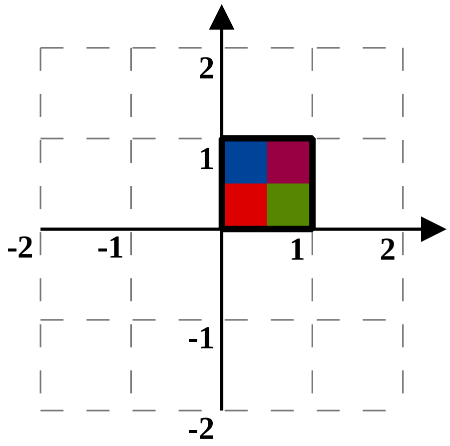
Description :



$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

	$\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$
$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 0 & & & \\ 0 & & & \end{pmatrix}$

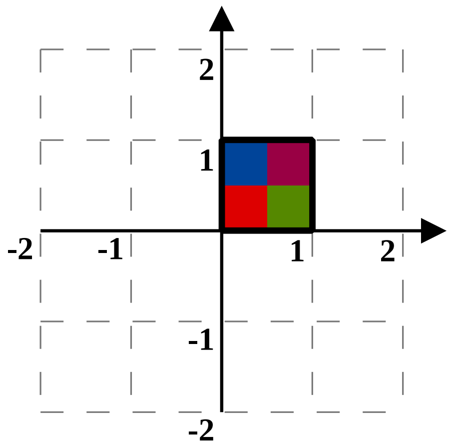
Description :



$$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

	$\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$
$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & & & \\ 0 & & & \end{pmatrix}$

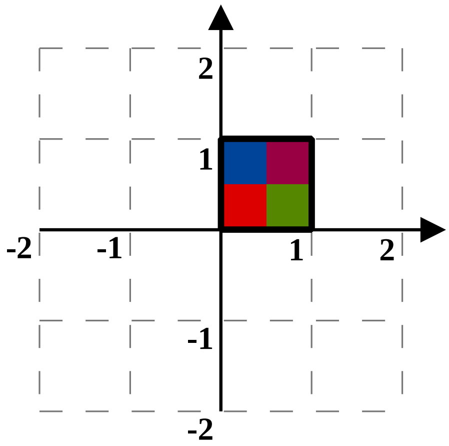
Description :



$$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

	$\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$
$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$

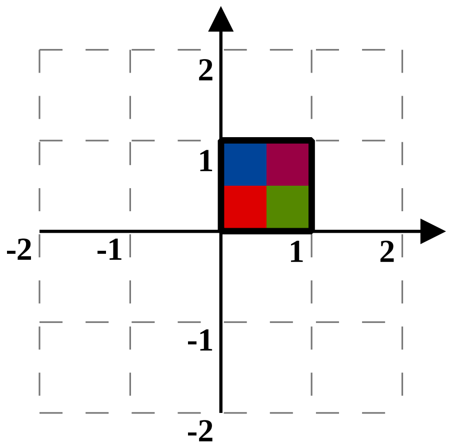
Description :



$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

	$\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$
$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$

Description :



Question 3

Further A-Level Examination Question from May 2016, FP1, Q1

Given that k is a real number and that,

$$\mathbf{A} = \begin{pmatrix} 1 + k & k \\ k & 1 - k \end{pmatrix}$$

find the exact values of k for which A is a singular matrix.

Give your answers in their simplest form.

[3 marks]

Question 4

Further A-Level Examination Question from January 2011, FP1, Q2

$$\mathbf{A} = \begin{pmatrix} 2 & 0 \\ 5 & 3 \end{pmatrix}, \qquad \mathbf{B} = \begin{pmatrix} -3 & -1 \\ 5 & 2 \end{pmatrix}$$

(a) Find **AB**

[3 marks]

Given that,

$$\mathbf{C} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

(b) describe fully the geometrical transformation represented by **C**

[2 marks]

(c) write down C^{100}

[1 mark]

Question 5

Further A-Level Examination Question from June 2011, FP1, Q3

(a) Given that $\mathbf{A} = \begin{pmatrix} 1 & \sqrt{2} \\ \sqrt{2} & -1 \end{pmatrix}$

(**i**) find $\mathbf{A}^2$

[3 marks]

(ii) describe fully the geometrical transformation represented by $\mathbf{A}^2$

[1 mark]

(b) Given that $\mathbf{B} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

describe fully the geometrical transformation represented by **B**

[2 marks]

(c) Given that $\mathbf{C} = \begin{pmatrix} k+1 & 12 \\ k & 9 \end{pmatrix}$

where k is a constant, find the value of k for which the matrix **C** is singular

[3 marks]