

Lesson 3

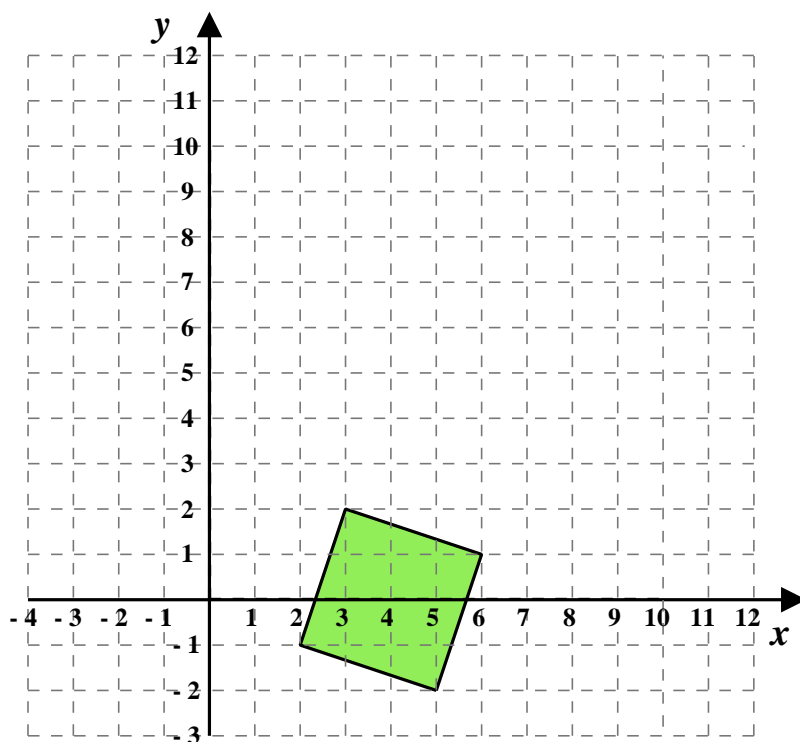
Further A-Level Pure Mathematics Matrix Transformations : Core 1

3.1 The Multipoint Matrix

A square has vertices $(3, 2)$, $(6, 1)$, $(5, -2)$ and $(2, -1)$

- (i) Write the shape as a multipoint matrix & transform it with $\mathbf{M} = \begin{pmatrix} 1 & 4 \\ 2 & -1 \end{pmatrix}$
- (ii) Add a plot of the transformed shape to the graph below.

Teaching Video : <http://www.NumberWonder.co.uk/v9090/3.mp4>



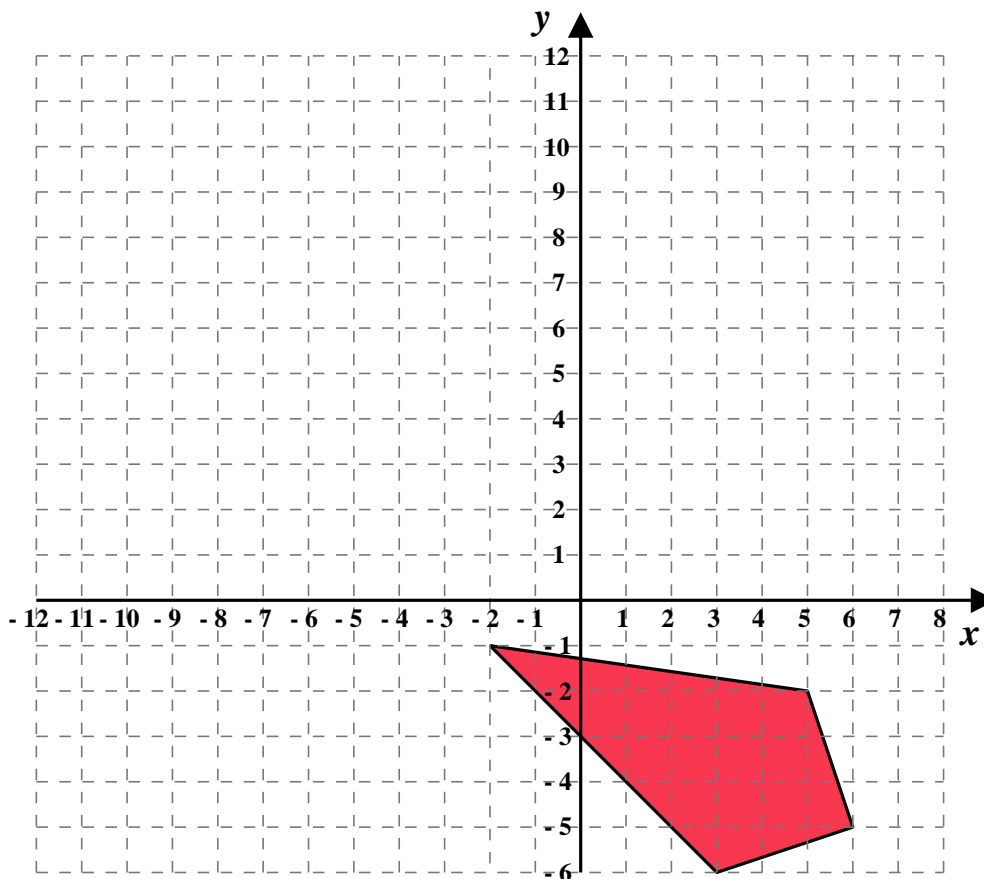
Note : The determinant of \mathbf{M} is a negative number, -9
The magnitude of $\det \mathbf{M}$ is 9 : the Area Scale Factor of the transformation.

3.2 Exercise

Question 1

A kite, **K**, has vertices $(5, -2)$, $(6, -5)$, $(3, -6)$ and $(-2, -1)$

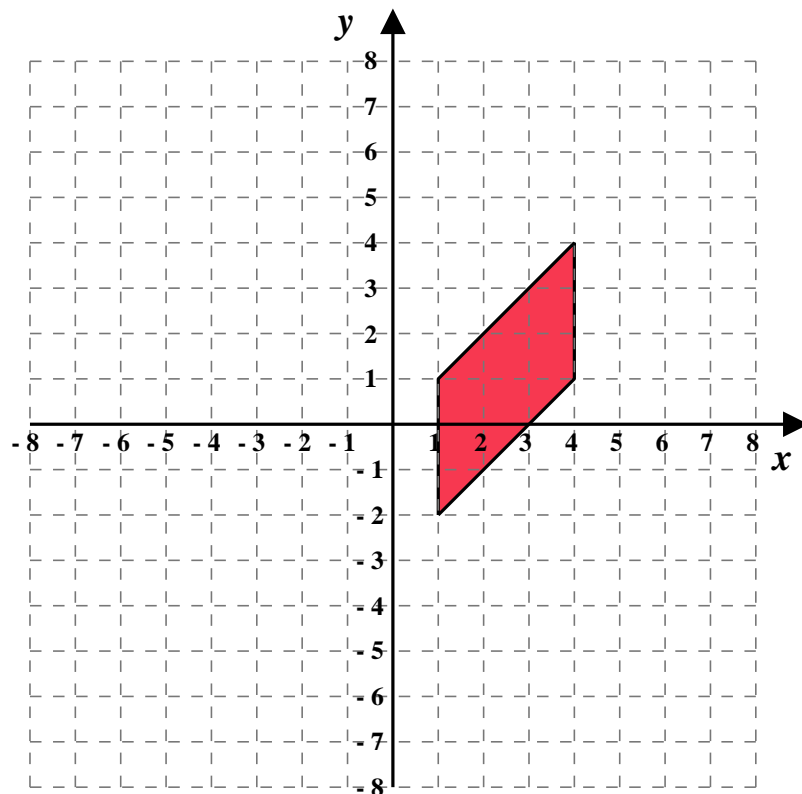
- (i) Write the shape as a multipoint matrix & transform it with $\mathbf{M} = \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$
- (ii) Add a plot of the transformed shape to the graph below.
- (iii) What is the area scale factor of the transformation ?



Question 2

A parallelogram has vertices $(1, -2)$, $(4, 1)$, $(4, 4)$ and $(1, 1)$

- (i) Write the shape as a multipoint matrix & transform it with $\mathbf{M} = \begin{pmatrix} 1 & -2 \\ -2 & 1 \end{pmatrix}$
- (ii) Add a plot of the transformed shape to the graph below.
- (iii) Find the magnitude of $\det \mathbf{M}$ and explain what it tells you about the transformation.



Question 3

This question is about working out the following matrix multiplication,

$$\begin{pmatrix} 5 & -1 & 6 \\ 8 & 3 & -4 \end{pmatrix} \times \begin{pmatrix} 2 & 9 & 1 & -6 \\ -3 & 12 & -5 & 7 \\ 4 & -2 & 0 & 11 \end{pmatrix}$$

Here is the multiplication grid already set up,

	$\begin{pmatrix} 2 & 9 & 1 & -6 \\ -3 & 12 & -5 & 7 \\ 4 & -2 & 0 & 11 \end{pmatrix}$
$\begin{pmatrix} 5 & -1 & 6 \\ 8 & 3 & -4 \end{pmatrix}$	$\begin{pmatrix} & 21 & & \\ & & & -71 \end{pmatrix}$

And here is how the 21 and the (-71) were found:

- In the product matrix, the 21 came from, $5 \times 9 + (-1) \times 12 + 6 \times (-2)$
- In the product matrix, the (-71) came from, $8 \times (-6) + 3 \times 7 + (-4) \times 11$

Complete the matrix multiplication grid.

Question 4

Further A-Level Examination Question from January 2015, IAL, F1, Q6 (ii)

$$\mathbf{M} = \begin{pmatrix} 2k + 5 & -4 \\ 1 & k \end{pmatrix} \text{ where } k \text{ is a real number}$$

Show that $\det \mathbf{M} \neq 0$ for all values of k

[4 marks]

Question 5

Further A-Level Examination Question from January 2014, IAL, F1, Q2

(i) $\mathbf{A} = \begin{pmatrix} -4 & 10 \\ -3 & k \end{pmatrix}$ where k is a constant

The triangle T is transformed to the triangle T' by the transformation represented by \mathbf{A}

Given that the area of triangle T' is twice the area of triangle T , find the possible values of k

[4 marks]

(ii) Given that,

$$\mathbf{B} = \begin{pmatrix} 1 & -2 & 3 \\ -2 & 5 & 1 \end{pmatrix} \quad \mathbf{C} = \begin{pmatrix} 2 & 8 \\ 0 & 2 \\ 1 & -2 \end{pmatrix}$$

find \mathbf{BC}

[3 marks]