3.1 Spin About Y

The volume of revolution formed when $x = f(y)$ is rotated through 2π radians about the y-axis between $y = a$ and $y = b$ is given by

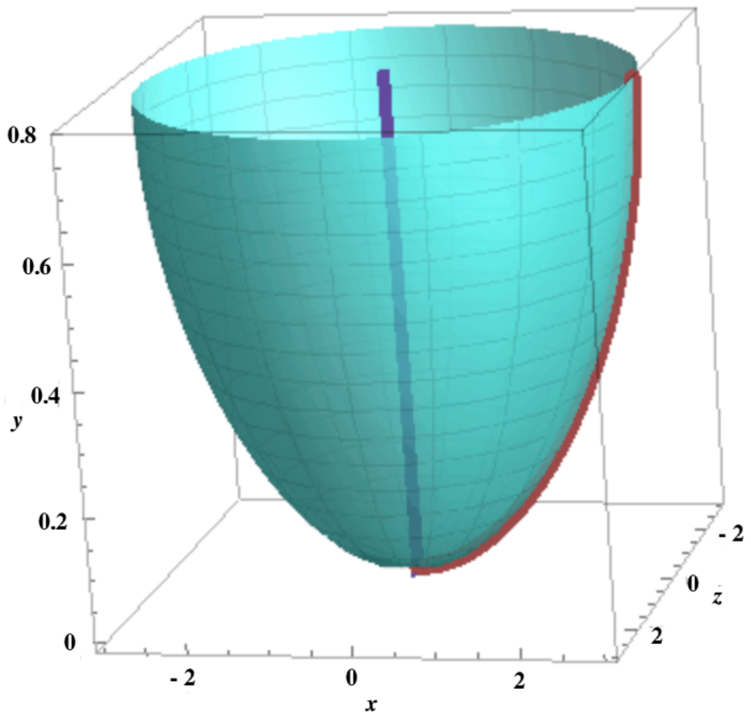
$$Volume = \pi \int_a^b x^2 \, dy$$

3.2.1 The Question

Find the exact volume swept out by the part of the following profile curve between the bounding lines given when it is rotated by $2\pi^c$ about the y-axis.

$$x = 3\sqrt{\sin(2y)}\,, \qquad y = 0, \qquad y = \frac{\pi}{4}$$

Give your answer as a multiple of π



You may like to try answering this question yourself before taking a look at the answer over the page.

NOTE : RADIANS must be used whenever trigonometry and calculus mix.

3.2.2 The Answer

$$
\begin{aligned}
Volume &= \pi \int x^2 \, dy \\
&= \pi \int_0^{\frac{\pi}{4}} 9 \sin(2y) \; dx \\
&= \frac{9\pi}{2} \int_0^{\frac{\pi}{4}} 2 \sin(2y) \; dx \qquad \text{Setting up a “”Chain ”Rule ”Backwards”} \\
&= \frac{9\pi}{2} \left[-\cos(2y)\right]_0^{\frac{\pi}{4}} \\
&= \frac{9\,\pi}{2} \left[-\cos\left(\frac{\pi}{2}\right) + \cos\;(0)\right] \\
&= \frac{9\,\pi}{2} \left[-0 + 1\right] \\
&= \frac{9\pi}{2}
\end{aligned}
$$

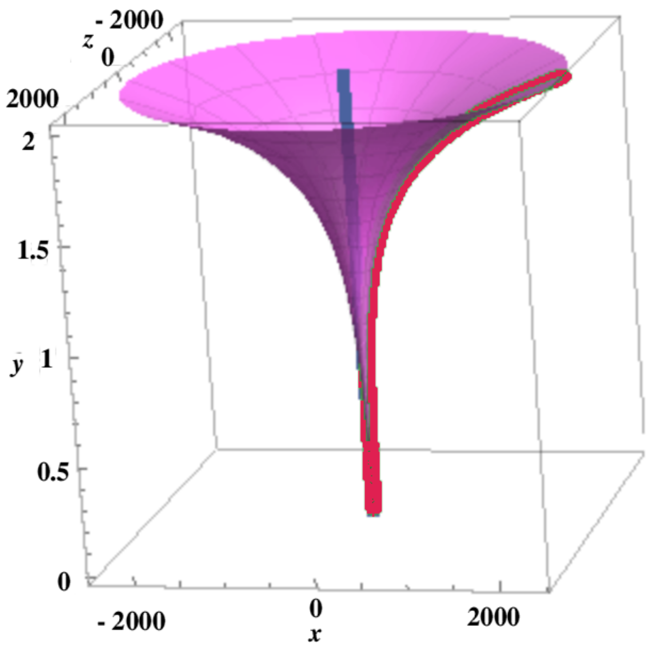
3.3 A Handy Table is Trigonometric Derivatives & Integrals

$f(x)$	$f'(x)$	
$\sin x$	$\cos x$	
$\cos x$	$-\sin x$	
$\tan x$	$\sec^2 x$	*
$\sec x$	$\sec x \; \tan x$	*
$\csc x$	$-\csc x \; \cot x$	*
$\cot x$	$-\csc^2 x$	*
$\ln\lvert x \rvert$	$\frac{1}{x}$	
$\ln\lvert \sec x \rvert$	$\tan x$	*
$\ln\lvert \sin x \rvert$	$\cot x$	*
$\ln\lvert \sec x + \tan x \rvert$	$\sec x$	*
$\ln\lvert \tan\left(\frac{1}{2}x + \frac{1}{4}\pi\right) \rvert$	$\sec x$	*
$-\ln\lvert \csc x + \cot x \rvert$	$\csc x$	*
$\ln\lvert \tan\left(\frac{1}{2}x\right) \rvert$	$\csc x$	*
e^x	e^x	

* Formulae marked with an asterix are provided in the examination in a book of formulae.

3.4 Exercise

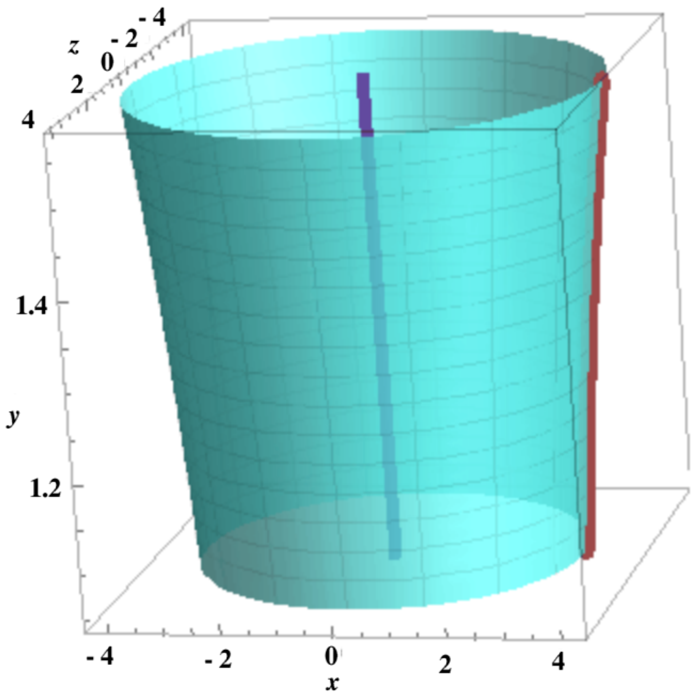
Question 1



(i) Show that the volume swept out by the curve $x = 6\,e^{3y}$ between $y = 0$ and $y = 2$ when it is rotated by $2\pi^c$ about the y-axis is exactly $6\,\pi\,(\,e^{12} - 1\,)$

(ii) Give this volume as a decimal correct to three decimal places.

Question 2



(i) Show that the volume swept out by the curve $x = \dfrac{3}{\cos(0.5y)}$ between $y = \dfrac{\pi}{3}$ and $y = \dfrac{\pi}{2}$ when it is rotated by $2\pi^c$ about the y-axis is exactly $6\pi\left(3 - \sqrt{3}\right)$

(ii) Give this volume as a decimal correct to three decimal places.

Question 3

The volume of revolution of a shot glass is 4π cm^3 exactly.
The profile curve is $x = \sqrt{y}$ and the rotation is about the y-axis.
The lower limit of the profile curve is $y = 1$ cm.
The upper limit is not known, call it a cm.
Calculate the upper limit, a, of the profile curve.
Clearly showing your method and working.

Question 4

Find the exact volume swept out by the part of the following profile curve between the bounding lines given when it is rotated by $2\pi^c$ about the y-axis.

$$x = y + \frac{1}{\sqrt{y}}, \qquad x = 1, \qquad x = 4$$

Write your answer in the form $\pi\,(K + \ln 4)$ where K is a constant, the value of which you should determine.

Question 5

(i) Use the product rule to differentiate with respect to y,

$$x = y \ln y$$

(ii) Hence show that,

$$\int_1^8 (1 + \ln y)\, dy = 24 \ln 2$$

(iii) Hence state the volume of the solid formed when the profile curve

$$x = \sqrt{1 + \ln y}$$

is rotated 2π radians about the y-axis between $y = 1$ and $y = 8$

Question 6

Show that the volume swept out by the curve

$$x = \frac{1}{4} e^{\frac{y}{2}}$$

between $y = 0$ and $y = 4 \ln 3$ when it is rotated by $2\pi^c$ about the y-axis is exactly 5π

Question 7

Find the volume swept out by the part of the following profile curve between the bounding lines given when it is rotated by $2\pi^c$ about the y-axis.

$$x = \frac{1}{3}\sqrt{sin\left(\frac{y}{2}\right)}, \qquad y = 0, \qquad y = \frac{\pi}{6}$$

Give your answer correct to 3 significant figures.