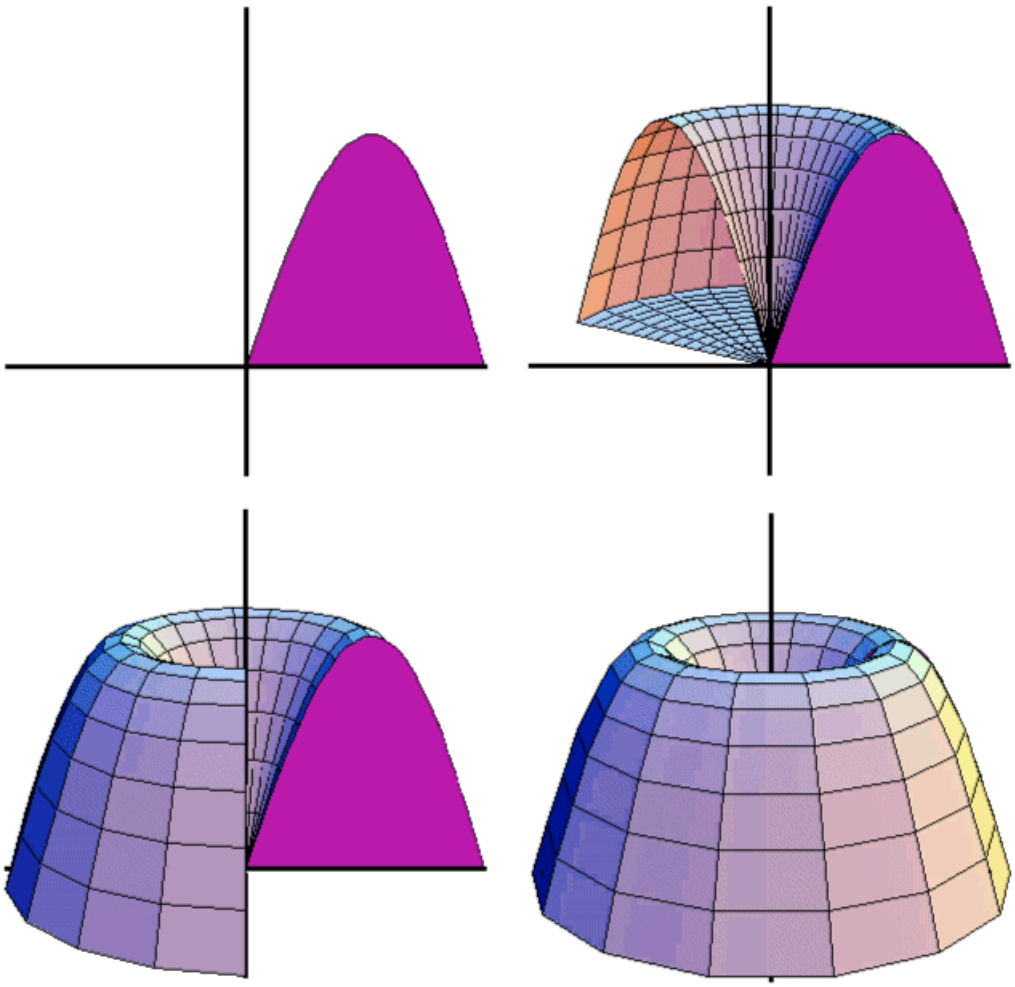
Further Pure A-Level Mathematics
Compulsory Course Component
Core 1

VOLUMES of REVOLUTION



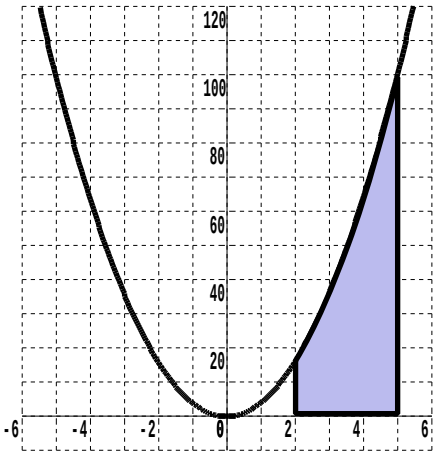
V O L U M E S of R E V O L U T I O N

Lesson 1

Further A-Level Pure Mathematics
Volumes of Revolution : Core 1

1.1 Introduction

Previously, integration was used to determine exactly the area under a curve.



Thus, for example, to find the area bounded by the curve $y = 4x^2$, the x-axis and the lines $x = 2$ and $x = 5$, we set up the following piece of mathematics;

$$
\begin{aligned}
Area &= \int_2^5 4x^2\,dx \\
&= \left[\frac{4x^3}{3}\right]_2^5 \\
&= \left[\frac{4\times 5^3}{3}\right] - \left[\frac{4\times 2^3}{3}\right] \\
&= \left[\frac{500}{3}\right] - \left[\frac{32}{3}\right] \\
&= \left[\frac{468}{3}\right] \\
&= 156
\end{aligned}
$$

Surprisingly, the mathematics to determine the volume swept out by this area as it rotates 360° about the x-axis is not much more complicated.
In an examination, the starting point is the following formula which should be memorised;

$$Volume = \pi \int y^2\,dx$$

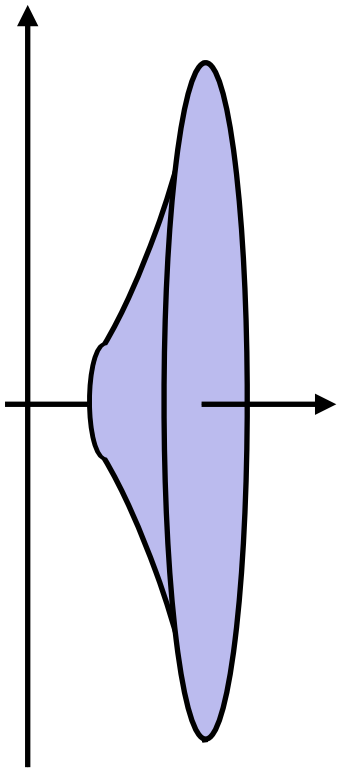
Where this comes from will be investigated shortly, but first let's master how to use it.

1.2 Example

1.2.1 The Question

Find the volume of the solid formed by rotating 360° about the x-axis the curve $y = 4x^2$ between $x = 2$ and $x = 5$.

Note : This is the same profile curve who's area was found in the introduction.



1.2.2 The Answer

$$
\begin{aligned}
Volume &= \pi \int y^2 \, dx \\
&= \pi \int_2^5 \left(4x^2\right)^2 dx \qquad \text{Common Error is forgetting to square the } y \\
&= \pi \int_2^5 16x^4 \, dx \\
&= 16\pi \int_2^5 x^4 \, dx \\
&= 16\pi \left[\frac{x^5}{5} \right]_2^5 \\
&= 16\pi \left\{ \left[\frac{5^5}{5} \right] - \left[\frac{2^5}{5} \right] \right\} \\
&= 16\pi \left[\frac{3093}{5} \right] \\
&= \frac{49488\,\pi}{5}
\end{aligned}
$$

which, if the units are cm, is about 31 000 cm^3

1.3 Exercise

Question 1

Find the volumes of the solids formed by rotating completely about the x-axis the areas bounded by the x-axis and the given curves and lines.

(i)

$$y = 5x^2, \qquad x = -1, \qquad x = 3$$

(ii)

$$y = 3x - x^2, \qquad x = 1, \qquad x = 2$$

(iii)

$$y = 2x - 5, \qquad x = 1, \qquad x = 4$$

(iv)

$$y = x^3 + 1, \qquad x = -1, \qquad x = 2$$

(v)

$$y = 1 + \sqrt{x}, \qquad x = 4, \qquad x = 9$$

(vi)

$$y = x + \frac{1}{x}, \qquad x = \frac{1}{2}, \qquad x = 2$$

Question 2

Find the volumes of the solids formed by rotating completely about the x-axis the areas enclosed by each of the following curves and the x-axis.

(i)

$$y = x^2 - 4$$

(ii)

$$3y = x^2 (3 - x)$$

Question 3

Explain where the Volume of Revolution formula comes from.
i.e.

$$Volume = \pi \int y^2 \, dx$$

HINT : Look up a textbook, search online...