Lesson 2

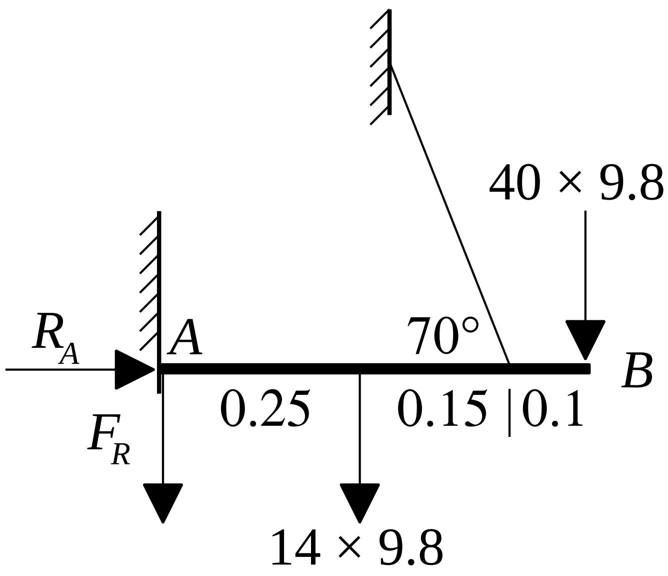
A-Level Applied Mathematics
Mechanics : Moments II : Year 2

2.1 Unhinged

A rock climber is abseiling down a rock face. On the way down she stops to have her photograph taken. For dramatic effect she takes her hands off the rope and hangs motionless in equilibrium. Although her boots are about to slip on the vertical section of rock face against which they press, a spectacular photograph is obtained.

At the moment the photograph is being taken, her situation can be modelled as follows,

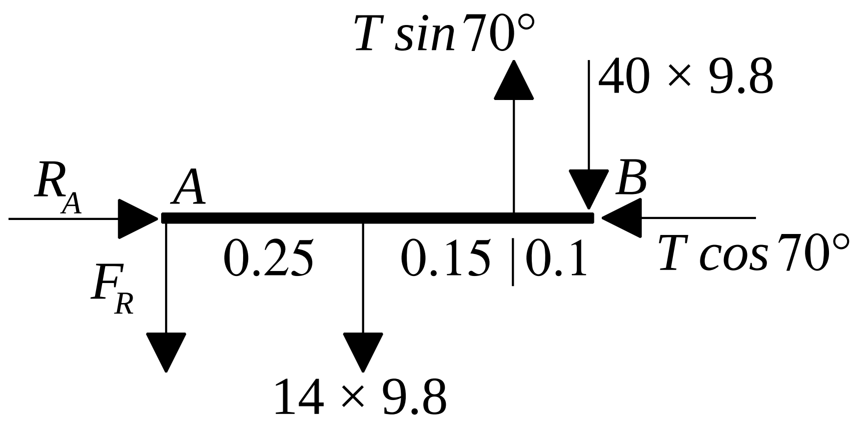
- Her legs as a single uniform horizontal rod *AB* of mass 14 kg and length 0.5 m
- The remainder of her body as a mass of 40 kg acting at *B*
- The supporting rope as being attached to the rod 0.1 m from end *B*
- The overall reaction of the rock on her shoes as being composed of two component parts, a reaction perpendicular to the vertical rock face, R_A, and a friction force parallel to the vertical rock face, F_R



The difference between this situation and those of Lesson 1 is that, rather than having a hinge at one end of the rod where is meets a vertical support, it is simply friction that is holding that end of the rod up.

The interest is in finding,
(i) The tension in the climber's rope, T,
(ii) The friction force, F_R, that's stopping her boots sliding up the rock face,
(iii) The normal reaction, R_A, on her boots,
(iv) The coefficient of friction, μ, between her boots and the rock face.
In order to relate this question to those of Lesson 1, the overall force exerted by the rock face on end A of the rod, her boots, will also be found.

To begin answering these questions, the following working diagram is the starting point. As before, the standard technique of resolving the tension in the climber's rope, T, has been applied to obtain its component parts, $T \sin 70°$ and $T \cos 70°$. Notice that the $T \cos 70°$ force has been translated along its "line of action" to place it at a convenient place on the diagram. That this can be done is a routine technique, used repeatedly in this topic.



Teaching Video : http://www.NumberWonder.co.uk/Video/v9084(2).mp4

(i) What is the tension in the climber's rope, T ?

(ii) Find the friction force, F_R, that's stopping her boots sliding up the rock face,

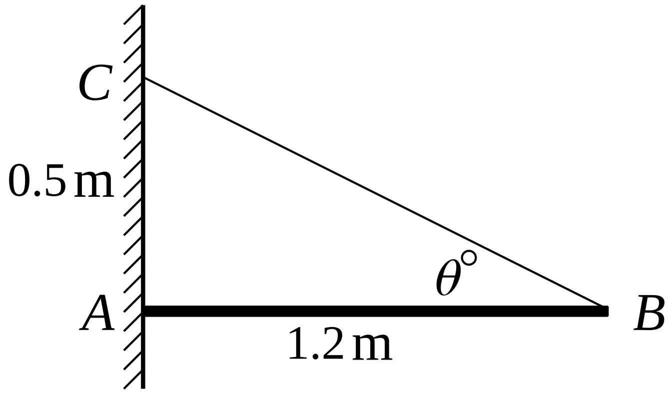
(iii) What is the normal reaction, R_A, on her boots ?

(iv) Determine coefficient of friction, μ, between her boots and the rock face.

(v) Calculate the overall force exerted by the rock face on end *A* of the rod.

2.2 Exercise

Question 1



A uniform rod *AB* of length 1.2 metres and mass 30 kg rests with one end, *A*, touching a rough horizontal wall. The rod is kept horizontal by a light inextensible string *BC* where *C* lies on the vertical wall directly above *A*. The tension in the tie *BC* is *T*. The plane *ABC* is perpendicular to the wall and ∠*ABC* is θ°.

(i) Draw a "working diagram" showing all significant forces acting on the rod including friction, F_R, parallel to the wall and a normal reaction, R_A, perpendicular to the wall. On this diagram show the tension, *T*, resolved into component parts parallel and perpendicular to the rod.

[4 marks]

(ii) Use simple trigonometry to write down the exact values of *tan* θ, *cos* θ and *sin* θ

[2 mark]

(iii) Show that the tension in the string is 39g Newtons.

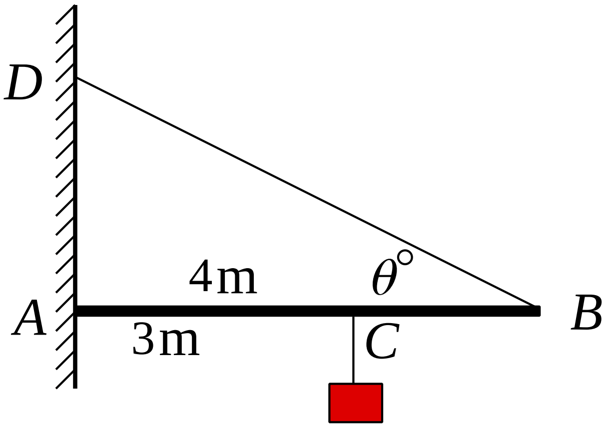
[4 marks]

(iv) Given that the rod is in limiting equilibrium, calculate the coefficient of friction between the rod and the wall.

[6 marks]

Question 2

A-Level examination Question from January 2007, M2, Q5 (edited)



A horizontal uniform rod *AB* has mass *m* and length 4 metres. The end *A* rests against a rough vertical wall. A particle of mass 2*m* is attached to the rod at the point *C*, where *AC* = 3 metres. One end of a light inextensible string *BD* is attached to the rod at *B* and the other end is attached to the wall at a point *D*, where *D* is vertically above *A*. The rod is in equilibrium in a vertical plane perpendicular to the wall.

The string is inclined at an angle θ to the horizontal, where $\tan\theta = \frac{3}{4}$, as shown.

(a) Find, in terms of *m* and *g*, the tension in the string.

[5 marks]

(b) Show that the horizontal component of the force exerted by the wall on the rod has magnitude $\frac{8}{3}mg$

[3 marks]

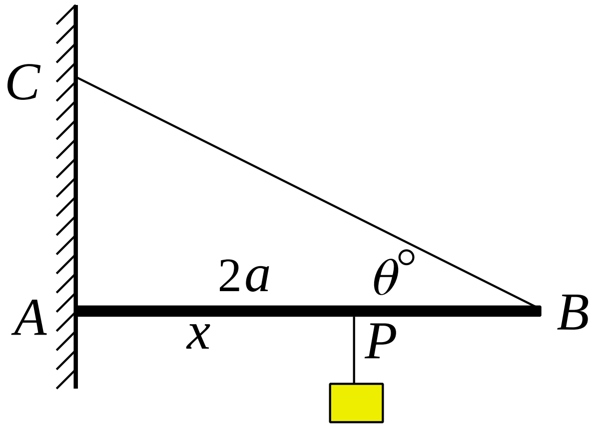
The coefficient of friction between the wall and the rod is μ. Given that the rod is in limiting equilibrium,

(c) find the value of μ

[4 marks]

Question 3

A-Level Examination Question from June 2018, Paper 3, Q9



A plank, *AB*, of mass *M* and length 2*a*, rest with its end *A* against a rough wall.
The plank is held in a horizontal position by a rope. One end of the rope is attached to the plank at *B* and the other end is attached to the wall at the point *C*, which is vertically above *A*.
A small block of mass 3*M* is attached to the plank at the point *P*, where *AP* = *x*.
The plank is in equilibrium in a vertical plane which is perpendicular to the wall.

The angle between the rope and the plank is θ, where $\tan\theta = \frac{3}{4}$, as shown.

The plank is modelled as a uniform rod, the block is modelled as a particle and the rope is modelled as a light inextensible string.

(a) Using the model, show that the tension in the rope is $\frac{5Mg(3x + a)}{6a}$

[3 marks]

The magnitude of the horizontal component of the force exerted on the plank at A by the wall is $2Mg$.

(b) Find x in terms of a.

[2 marks]

The force exerted on the plank at A by the wall acts in a direction which makes an angle β with the horizontal.

(c) Find the value of $\tan \beta$

[5 marks]

The rope will break if the tension in it exceeds $5Mg$.

(d) Explain how this will restrict the possible positions of P.
You must justify your answer carefully.

[3 marks]