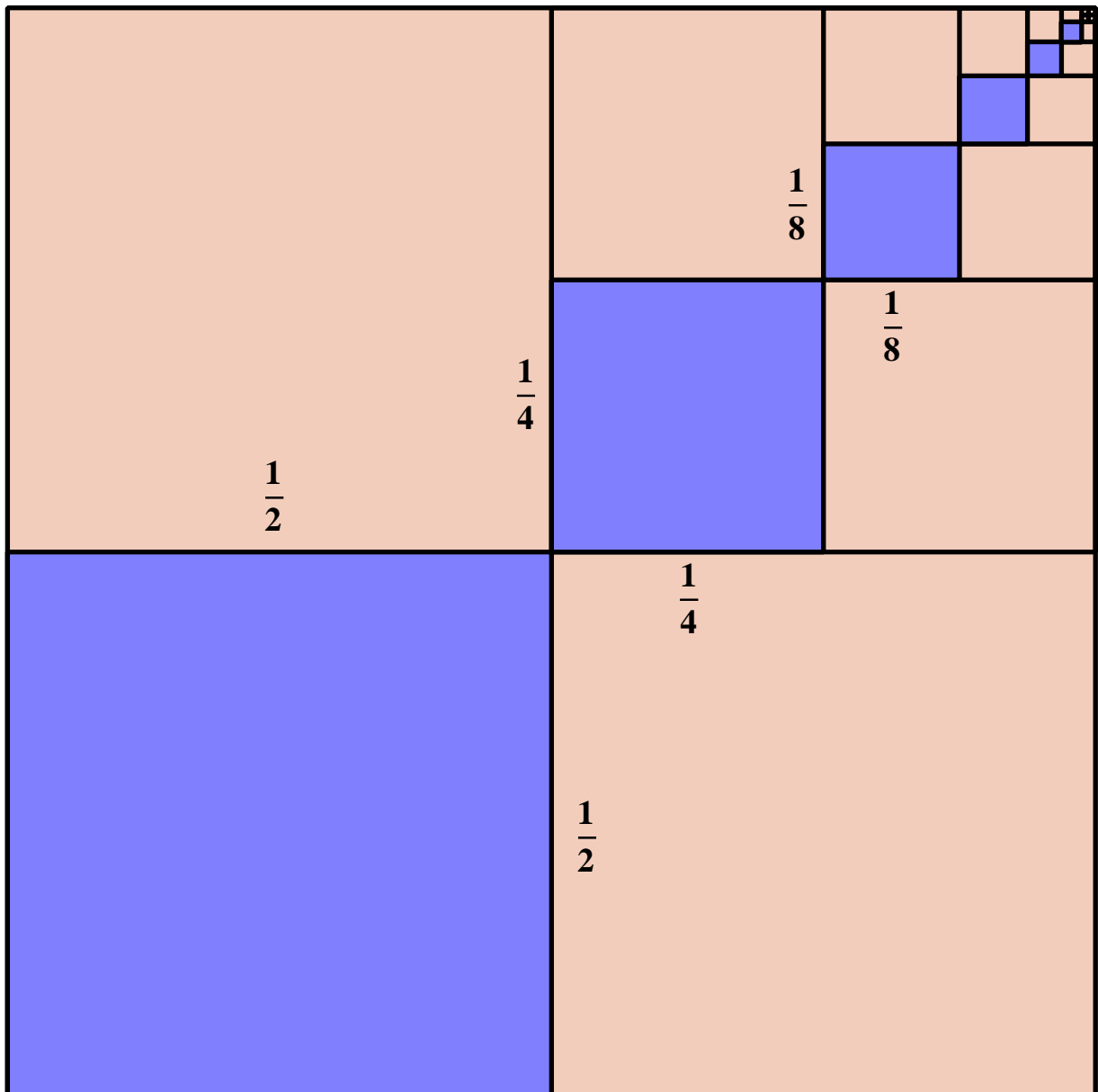


A-Level
~ Year 2 ~
Pure Mathematics

GEOMETRIC PROGRESSIONS



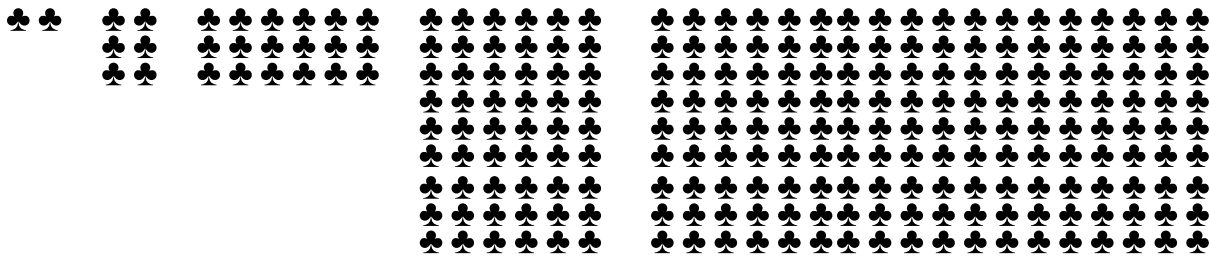
$$\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \frac{1}{256} + \frac{1}{1024} + \frac{1}{4096} + \dots = \frac{1}{3}$$

Geometric Progressions

Lesson 1

A-Level Pure Mathematics Geometric Progressions : Pure Year 2

1.1 How To Spot A Geometric Progression



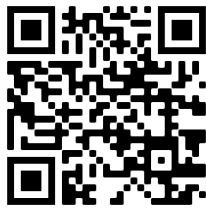
Consider the sum

$$2 + 6 + 18 + 54 + 162 + \dots$$

Explain why this series not an Arithmetic Progression



Teaching Video : <http://www.NumberWonder.co.uk/v9077/1.mp4>



Observe that the terms are linked; each is three times the previous

This is the hallmark of a Geometric Progression.

In this case it is said that the common ratio is 3.

Expressed algebraically, a Geometric Progression is of the form

$$a, ar, ar^2, ar^3, ar^4, \dots$$

where a is the first term

and r is the common ratio

Write down a formula for the n^{th} term, G_n , of a Geometric Progression



1.2 Example

The 5th term of a Geometric Progression is 567 and the 2nd term is 21.

(i) What is the common ratio ?

(ii) Write out the first 6 terms of the Geometric Progression.

(iii) Determine the exact value of the 20th term.

1.3 Exercise

Question 1

Write out the first five terms of the Geometric Progression with first term 8 and common ratio 1.5

Question 2

Write out the first five terms of the Geometric Progression with first term 8 and common ratio 0.5

Question 3

Write out the first five terms of the Geometric Progression with first term 3 and common ratio -2

Question 4

Consider the following Geometric Progression;

$$0.3, 0.03, 0.003, 0.0003, \dots$$

- (i) State the value of the first term, a , and the value of the common ratio, r

The sum of this Geometric Progression has an infinite number of terms

$$0.3 + 0.03 + 0.003 + 0.0003 + \dots$$

This infinite sum has a finite answer.

- (ii) Give the exact value of this 'sum to infinity'

Question 5

What is the exact value of the 20th term of the following geometric Progression ?

$$5, 15, 45, 135, \dots$$

Question 6

The 5th term of a Geometric Progression is 3750 and the 2nd term is 30.

- (i) What is the common ratio ?

- (ii) Write out the first 6 terms of the Geometric Progression.

- (iii) Determine the exact value of the 12th term.

Question 7

The 6th term of a Geometric Progression is 0.375 and the 3rd term is -3

(i) What is the common ratio ?

(ii) Write out the first 6 terms of the Geometric Progression

(iii) Determine the exact value of the 20th term

Write your answer as a $\frac{p}{q}$ fraction, for integer p and q

Question 8

For each of the following series state if the terms are in

- Arithmetic Progression
- Geometric Progression
- Neither Arithmetic nor Geometric Progression

(i) $7 + 3 - 1 - 5 - \dots$

(ii) $1 + 8 + 27 + 64 + \dots$

(iii) $0.1^3 + 0.1^5 + 0.1^7 + 0.1^9 + \dots$

(iv) $3 - 1 + \frac{1}{3} - \frac{1}{9} + \dots$

(v) $1 - 1 + 1 - 1 + 1 - 1 + \dots$

Question 9

Determine the value of this series which is in Geometric Progression, and expressed in sigma notation

$$\sum_1^4 3^n$$

Question 10

Determine the value of this series which is in Geometric Progression, and expressed in sigma notation

$$\sum_1^5 3 \times 2^n$$

Question 11

If 3, x and 9 are the first three terms of a sequence in Geometric Progression, find

(i) the possible exact values of x

(ii) the possible exact values of the 4th term

Question 12

The 7th term of a Geometric Progression is exactly 1.9487171
and the 3th term is exactly 1.331

(i) What is the common ratio ?

(ii) Write out the first 6 terms of the Geometric Progression.

(iii) Express the sum of first 40 terms of this Geometric Progression in
sigma notation.

Question 13

A geometric sequence has first term 4 and third term 1.

Find the two possible values of the 6th term

Question 14

The first three terms of a geometric sequence are given by

$$8 - x, \quad 2x, \quad x^2$$

respectively where $x > 0$

(i) Show that $x^3 - 4x^2 = 0$

[2 marks]

(ii) Find the value of the 20th term

[3 marks]

(iii) State, with a reason, whether 4096 is a term in the sequence

[1 mark]