## A-Level Applied Mathematics

Year 2<br>Mechanics

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## Lesson 1

## A-Level Applied Mathematics

Mechanics: Moments : Year 2

### 1.1 The Moment Of A Force

In Mechanics we often assume that the forces under consideration act upon a point which is typically described as being a particle.


However, in the physical world forces act on "lumps of stuff".


This can result in a "lump of stuff" rotating.

In this course we are primarily concerned with either preventing such rotation or in using the fact that an object is not rotating to deduce information about some of the forces applying.

The "lumps of stuff" are called rigid bodies and the turning effect of a force about a pivot is termed a moment. If an object is described as being in equilibrium then the overall turning effect upon it must be zero. We say that "the sum of the moments is zero" or equivalently that "the sum of the clockwise moments must equal the sum of the anti-clockwise moments".
(about any given pivot)

A seesaw makes use of moments and, in particular, the fact that a lighter person, a child, can approximately balance a heavier person, its mother, if the lighter person is further away from the pivot that the heavier person.


The sea-saw makes it clear that moments depend upon two factors;
(i) The magnitude of the force applied
( ii ) The distance of that force from the pivot

The mathematical relationship is as simple as could be hoped for;

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\text { moment }=\text { force applied } \times \text { distance from pivot }
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### 1.2 Example

Robert and George are trying to push in opposite directions through a door. Robert applies a force of 50 N at a distance of 0.6 m from the door's hinge.
George applies a force of 70 N at a distance of 0.4 m from the door's hinge.

A bird's eye view from above the door is shown.


Who will win this pushing battle ?
Justify your answer.
Teaching video : http://NumberWonder.co.uk/Video/v9069(1a).mp4

### 1.3 Example

A lever consists of a uniform steel rod $A B$ of mass 6 kg and length 3 m , which rests on a small smooth pivot at a point $C$. A load of mass 8 kg is suspended from the end $B$ of the rod by a light rope. The lever is held in equilibrium in a horizontal position by a vertical force applied at end $A$, as shown in the diagram. Given that $B C=0.75 \mathrm{~m}$ find the magnitude of the force, $F_{A}$, applied at $A$.


Teaching Video : http://www.NumberWonder.co.uk/Video/v9069(1b).mp4

### 1.4 Exercise

## Question 1

Emma, Lucy and Lucy's friend Tracy (who's not very bright) are trying to push in opposite directions through a door.
Emma applies a force of 70 N at a distance of 0.8 m from the door's hinge.
Lucy applies a force of 60 N at a distance of 0.7 m from the door's hinge assisted by Tracy who applies a force of 50 N at a distance of 0 m from the door's hinge.

A bird's eye view from above the door is shown.

(i) Who will win this pushing battle, Emma or Lucy \& Tracy ? Justify your answer.
( ii ) If Emma and Lucy push as before, where would Tracy have to push in order for the door to be in equilibrium?

## Question 2

A lever consists of a uniform steel rod $A B$ of mass 7 kg and length 4 m , which rests on a small smooth pivot at a point $C$. A load of mass 12 kg is suspended from the end $B$ of the rod by a light rope. The lever is held in equilibrium in a horizontal position by a vertical force applied at end $A$, as shown in the diagram.
Given that $B C=1.25 \mathrm{~m}$ find the magnitude of the force, $F_{A}$, applied at $A$.


## Question 3

A large Christmas decoration is made from a uniform rod, $A B$, of mass 3 kg and length 3.5 metres.
It is suspended at $C$ where length $A C$ is 1.5 metres.
Cut out stars hang from the rod each of mass $m$; Four stars at $A$, one from $C$ and two from $B$.
Given that the decoration is in equilibrium, calculate the mass of a star, $m$


## Question 4

A-Level Examination Question from January 2004, M1, Q2


A lever consists of a uniform steel rod $A B$, of weight 100 N and length 2 m , which rests on a small smooth pivot at a point $C$ of the rod. A load of weight 2200 N is suspended from the end $B$ of the rod by a rope. The lever is held in equilibrium in a horizontal position by a vertical force applied at the end $A$, as shown. The rope is modelled as a light string.
Given that $B C=0.2 \mathrm{~m}$,
( a ) find the magnitude of the force applied at $A$

The position of the pivot is changed so that the rod remains in equilibrium when the force at $A$ has magnitude 120 N .
(b) Find, the the nearest cm , the new distance of the pivot from $B$.

