

# Grade Grabber 5

*Any solution based entirely on graphical  
or numerical methods is not acceptable*

Marks Available : 30

## **Question 1**

The gradient of a curve at any point  $(x, y)$  on the curve is inversely proportional to the product of  $x$  and  $y$ . The curve passes through the point  $(1, 1)$  and at this point the gradient of the curve is 7.

Form a differential equation in  $x$  and  $y$  and solve this equation to express  $y^2$  in terms of  $x$

**Question 2**

Relative to a fixed origin  $O$ , the points  $P$  and  $Q$  are such that

$$\vec{OP} = \begin{pmatrix} 7 \\ 12 \\ -1 \end{pmatrix} \quad \vec{OQ} = \begin{pmatrix} 11 \\ 2 \\ a \end{pmatrix}$$

where  $p$  is a constant, and the points  $R$  and  $S$  are such that

$$\vec{QR} = \begin{pmatrix} 3 \\ -16 \\ 12 \end{pmatrix} \quad \vec{PS} = \begin{pmatrix} 1 \\ -11 \\ 16 \end{pmatrix}$$

( i ) Find the position vector of the point  $S$

[ 2 marks ]

( ii ) Find  $\vec{OR}$  in term of  $a$

[ 2 marks ]

( iii ) Find  $\vec{SR}$  in term of  $a$

[ 2 marks ]

( iv ) Find  $\vec{PQ}$  in term of  $a$

[ 2 marks ]

( v ) Find the value of  $a$

[ 3 marks ]

**Question 3**

Two trigonometry identities involving  $\cos^2 \theta$  and  $\sin^2 \theta$  are;

$$\cos^2 \theta + \sin^2 \theta = p$$

$$\cos^2 \theta - \sin^2 \theta = q$$

where  $p$  is a constant and  $q$  is a trigonometric function in  $\theta$

- (i) Write out the two identities with  $p$  and  $q$  replaced appropriately

[ 2 marks ]

- (ii) Treating the two identities as simultaneous equations, combine them by addition to obtain an expression for  $\cos^2 \theta$

[ 2 mark ]

- (iii) Hence, or otherwise, evaluate

$$\int_0^{\frac{\pi}{2}} \cos^2 \theta \, d\theta$$

[ 3 marks ]

**Question 4**

Given that

$$x = -3 \cot y \quad -\frac{\pi}{2} < y < \frac{\pi}{2}$$

show that

$$\frac{dy}{dx} = \frac{a}{x^2 + b}$$

where  $a$  and  $b$  are integer constants to be found**[ 5 marks ]**

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Teachers may obtain detailed worked solutions to the exercises by email from [mhh@shrewsbury.org.uk](mailto:mhh@shrewsbury.org.uk)