A-Level Pure Mathematics
Year 1, Year 2 Revision

Grade Grabber 4

Any solution based entirely on graphical
or numerical methods is not acceptable

Marks Available : 30

Question 1

(i) Show that

$$\frac{\cos x}{1 - \sin x} + \frac{1 - \sin x}{\cos x} = k \sec x \qquad x \neq \frac{n\pi}{2}, \ n \in Z$$

where k is a constant to be found

[4 marks]

(ii) Hence explain why the equation

$$\frac{\cos x}{1 - \sin x} + \frac{1 - \sin x}{\cos x} = 0.8$$

has no real solutions

[2 marks]

Question 2

A curve has parametric equations $x = 6 \sin t \qquad y = 5 \cos 2t$

Find the Cartesian equation of the curve in the form $y = ax^2 + bx + c$ where *a*, *b* and *c* are real number constants.

[4 marks]

Question 3

The function *f* is defined by

$$f(x) = \frac{6x}{2x + 3} \qquad x \in \mathbb{R},\ x \geqslant 0$$

Show that

$$ff(x) = \frac{4x}{2x + 1}$$

[4 marks]

Question 4
The gradient of a curve at any point (x, y) on the curve is directly proportional to the product of x and y. The curve passes through the point (1, 1) and at this point the gradient of the curve is 4.

Form a differential equation in x and y and solve this equation to express y in terms of x

[8 marks]

Question 5

$$y = \tan x \ \sec^{16} x$$

This question is about finding the integral of *y* in two different ways.

(i) Use the fact that

$$\int y\,dx = \int (\sec x \ \tan x)\ (\sec x)^{15}\ dx$$

and "the chain rule backwards" to determine an algebraic answer for the integration

[4 marks]

(ii) Show that

$$\int y\,dx = (-1)\int (-\sin x)\ (\cos x)^{-17}\ dx$$

and hence, by means of "the chain rule backwards", determine an algebraic answer for the integration. Manipulate your anwers to show that it is the same as that obtained in part (i).

[4 marks]