Lesson 4

A-Level Pure : The Binomial Theorem : Year 2

4.1 The Binomial Theorem & Partial Fractions

Earlier work with Partial Fractions allows us to cope with finding the Binomial Expansion of more difficult expressions.

4.2 Example

$$f(x) = \frac{x^2 + 3}{(1 - x)(1 + x)^2} \qquad x \neq \pm 1$$

Express $f(x)$ as partial fractions, and so obtain a cubic approximation for $f(x)$
State the range of values of x for which the expansion is valid

4.3 Exercise

Question 1

$$g(x) = \frac{1}{(1 + 3x)(1 + 2x)} \qquad x \neq -\frac{1}{3}, \quad -\frac{1}{2}$$

Express $g(x)$ as partial fractions, and so obtain a cubic approximation for $g(x)$
State the range of values of x for which the expansion is valid

Question 2

$$h(x) = \frac{14 + 5x}{(1 + x)(2 - x)} \qquad x \neq -1,\ 2$$

Express $h(x)$ as partial fractions, and so obtain a cubic approximation for $h(x)$. State the range of values of x for which the expansion is valid.

Question 3

C4 Examination Question from June 2006, Q2

$$f(x) = \frac{3x - 1}{(1 - 2x)^2} \qquad |x| < \frac{1}{2}$$

Given that, for $x \neq \frac{1}{2}$

$$\frac{3x - 1}{(1 - 2x)^2} = \frac{A}{(1 - 2x)} + \frac{B}{(1 - 2x)^2}$$

where A and B are constants,

(a) find the values of A and B

[3 marks]

(b) Hence, or otherwise, find the series expansion of $f(x)$ in ascending powers of x up to and including the term in x^3 simplifying each term

[6 marks]

Question 4

C4 Examination Question from January 2009, Q3

$$f(x) = \frac{27x^2 + 32x + 16}{(3x + 2)^2(1 - x)} \qquad |x| < \frac{2}{3}$$

Given that $f(x)$ can be expressed in the form

$$f(x) = \frac{A}{(3x + 2)} + \frac{B}{(3x + 2)^2} + \frac{C}{(1 - x)}$$

(a) find the values of B and C and show that $A = 0$

[4 marks]

(b) Hence, or otherwise, find the series expansion of $f(x)$ in ascending powers of x up to and including the term in x^2 simplifying each term

[6 marks]

(c) Find the percentage error made in using the series expansion in part **(b)** to estimate the value of $f(0.2)$. Give your answer to 2 significant figures.

[4 marks]