Lesson 2

Further A-Level Mathematics, Mechanics 1
Momentum and Impulse

2.1 A Second Formula For Impulse

Previously, Impulse was described as a *kick* or a *jerk* and we could measure its effect upon an object by measuring the change of momentum of the object.

$$I = mv - mu$$

Intuitively, to stop an oil tanker using a tug, (i.e. To take away its momentum) a tug needs to apply a stopping *force* over a sustained period of *time*. The tug is applying a slow motion, gentle, *kick* to the oil tanker. (So as not to damage it) This suggests that there is another formula for Impulse that will involve force and time, and this is indeed the case.

$$\begin{aligned} I &= mv - mu \\ &= m(v - u) \qquad But \qquad v = u + at \\ &\qquad\qquad\quad so \quad v - u = at \\ &= m\,a\,t \\ &\qquad\qquad\quad But \qquad F = ma \\ &= F\,t \end{aligned}$$

Thus we have a second formula for Impulse;

$$Impulse = Force \times Time$$

Notice that the units of Impulse can be either kg ms^{-1} or Ns

2.2 Example N° 1

A toy train is pushed along a straight track by a horizontal force of 3 N for 4 s. Find the gain in momentum of the train.

2.3 Example N° 2

M1 Exam question, 12th November 2007, Q4

A particle P of mass 0.3 kg is moving with speed u ms^{-1} in a straight line on a smooth horizontal table. The particle P collides directly with a particle Q of mass 0.6 kg, which is at rest on the table. Immediately after the particles collide, P has speed 2 ms^{-1} and Q has speed 5 ms^{-1}. The direction of motion of P is reversed by the collision.

Find

(a) the value of u

[4 marks]

(b) the magnitude of the impulse exerted by P on Q

[2 marks]

Immediately after the collision, a constant force of magnitude R newtons is applied to Q in the direction directly opposite to the direction of motion of Q. As a result Q is brought to rest in 1.5 s

(c) Find the value of R

[4 marks]

2.4 Exercise

Question 1

A particle of mass 12 kg is initially moving with velocity 3 ms^{-1} on a smooth horizontal surface. A 4 N horizontal force acts on the particle for 6 seconds. Find the final speed of the particle by using the fact that;

$$Ft = mv - mu$$

Question 2

A canon of mass 800 kg discharges a canonball of mass 20 kg.
The canon recoils against a constant force of 4 kN which brings it to rest in 1.25 s.
Find the speed of the shot.

Question 3

A canonball of mass 15 kg is fired with speed 560 ms^{-1}.
Given that the shell is in the barrel of the canon for 0.05 s, calculate the average force, in kN, exerted on the shell by the explosive charge.

Question 4

A concrete block of mass 2 kg falls from rest from a vertical height of 10 m above horizontal ground.

(a) Calculate the speed immediately before it hits the ground

The ground is soft and, after the concrete block reaches the ground, it sinks vertically downwards into the ground before coming to rest. The ground is assumed to exert a constant resistive force of magnitude 5600 N on the block.

(b) Find the vertical distance that the concrete block sinks into the ground before coming to rest. Give your answer in cm.

Question 5

M1 Exam question, 16th May 2012, Q5

A particle P is projected vertically upwards from a point A with speed u ms^{-1}. The point A is 17.5 m above horizontal ground. The particle P moves freely under gravity until it reaches the ground with speed 28 ms^{-1}.

(a) Show that $u = 21$

[3 marks]

At time t seconds after projection, P is 19 m above A.

(b) Find the possible values of t.

[5 marks]

The ground is soft and, after P reaches the ground, P sinks vertically downwards into the ground before coming to rest. The mass of P is 4 kg and the ground is assumed to exert a constant resistive force of magnitude 5000 N on P

(c) Find the vertical distance that P sinks into the ground before coming to rest.

[4 marks]