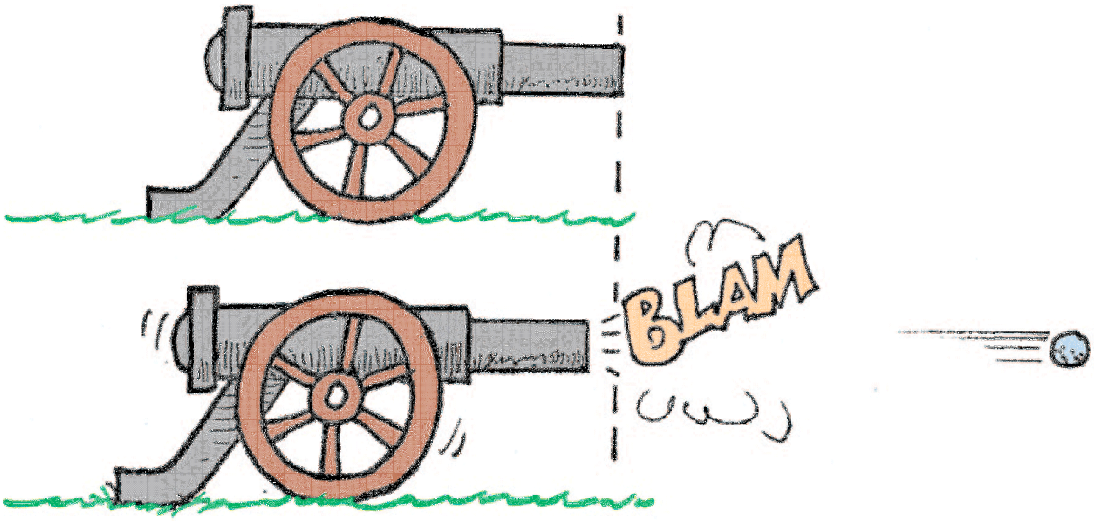
Further Mechanics A-Level Mathematics
Optional Course Component
Further Mechanics 1

MOMENTUM
&
IMPULSE



M O M E N T U M & I M P U L S E

Lesson 1

Further A-Level Mathematics, Mechanics 1

Momentum and Impulse

1.1 When Particles Collide

In this topic we consider what happens when particles collide at velocities well below the speed of light. The key idea is to look at the total momentum of the system before and, separately, after a collision has taken place.
Although, typically, energy is lost† in such collisions, the total momentum before and after the collision is the same.

We say that “momentum is conserved”

1.2 The Theory

Definition of Momentum

$$momentum = mass \times velocity$$

The Conservation of Momentum Principal

$$total\ momentum\ before\ collision = total\ momentum\ after\ collision$$

Kinetic Energy (the energy of movement)

$$KE = \frac{1}{2} m v^2$$

Impulse

Although the total momentum of the system just before and just after a collision is conserved, for an individual particle it changes.
The change, called the Impulse, can be thought of as a 'tug', 'kick' or 'jerk'.

$$Impulse = change\ in\ momentum$$

$$I = mv - mu$$

Where m is the mass of the individual particle (which will not change),
u is the velocity of that particle before the collision
and v is the velocity of that particle after the collision

† You could argue that energy is conserved but in a collision energy gets converted to sound, light, static charge and also does work in deforming the objects being modelled by the particles in the collision.
It is impractical to a precise keep track of it all - and so the difference between the Kinetic Energy before and after the collision is referred to as the energy lost. This is a somewhat misleading expression as a principal of physics is that energy can neither be created nor destroyed, although it can transform into mass. ($E = m c^2$)

1.3 Example

Balls of mass 6 kg and 4 kg are moving directly towards each other with speeds of 7 ms^{-1} and 2 ms^{-1} respectively.
They collide.
The direction of the 6 kg mass is unchanged by the collision, it's speed then being reduced to 4 ms^{-1}

(i) Find the speed of the 4 kg mass after the collision.

(ii) Determine the magnitude of the Impulse on the 4 kg mass.

(iii) How much Kinetic Energy (if any) has been lost in the collision ?

1.4 Exercise

Question 1

A small smooth sphere *A* of mass 2.5 kg lies at rest on a smooth horizontal table. A second small smooth sphere *B* of mass 1.5 kg is moving with speed 4 ms^{-1} and collides directly with *A*. The two spheres *coalesce* - in other words, after impact they move as a single body. Find their speed after impact.

Question 2

M1 Exam question, 12th January 2005, Q1

A particle *P* of mass 1.5 kg is moving along a straight horizontal line with speed 3 ms^{-1}. Another particle *Q* of mass 2.5 kg is moving, in the opposite direction, along the same straight line with speed 4 ms^{-1}.
The particles collide.
Immediately after the collision the direction of motion of *P* is reversed and its speed is 2.5 ms^{-1}.

(a) Calculate the speed of *Q* immediately after the impact.

[3 marks]

(b) State whether or not the direction of motion of *Q* is changed by the collision.

[1 mark]

(c) Calculate the magnitude of the impulse exerted by *Q* on *P*, giving the units of your answer.

[3 marks]

Question 3

M1 Exam question, 3rd June 2015, Q1

Particle P of mass m and particle Q of mass km are moving in opposite directions on a smooth horizontal plane when they collide directly. Immediately before the collision the speed of P is $5u$ and the speed of Q is u. Immediately after the collision the speed of each particle is halved and the direction of motion of each particle is reversed.

Find

(a) the value of k

[3 marks]

(b) the magnitude of the impulse exerted on P by Q in the collision

[3 marks]

Question 4

M1 Exam question, 5th November 2002, Q6

A railway truck *P* of mass 1500 kg is moving on a straight horizontal track. The truck *P* collides with a truck *Q* of 2500 kg at a point *A*.
Immediately before the collision, *P* and *Q* are moving in the same direction with speeds of 10 ms^{-1} and 5 ms^{-1} respectively.
Immediately after the collision, the direction of motion of *P* is unchanged and its speed is 4 ms^{-1}.

By modelling the trucks as particles,

(a) show that the speed of *Q* immediately after the collision is 8.6 ms^{-1}.

[3 marks]

After the collision at *A*, the truck *P* is acted upon by a constant braking force of magnitude 500 N. The truck *P* comes to rest at the point *B*.

(b) Find the distance *AB*.

[3 marks]

After the collision *Q* continues to move with constant speed 8.6 ms^{-1}.

(c) Find the distance between *P* and *Q* at the instant when *P* comes to rest.

[5 marks]