2.1 The Sum of a famous Arithmetic Progression

One of Germany's finest mathematicians, Carl Friedrich Gauss, [1777-1855] when young, was asked to sum the first 100 integers.
i.e. What is 1 + 2 + 3 = 4 + ... + 98 + 99 + 100 ?

Instantly, Gauss had the answer.
More than that, he had an interesting mental method.

	S =	1	+ 2	+ 3	+ 4	+ ...	+ 98	+ 99	+ 100
ADD	S =	100	+ 99	+ 98	+ 97	+ ...	+ 3	+ 2	+ 1
	2S =	101	+ 101	+ 101	+ 101	+ ...	+ 101	+ 101	+ 101

$\therefore$ 2S = 101 × 100
2S = 10100
S = 5050

2.2 The Sum of any Arithmetic Progression

Algebraically, an Arithmetic Progression is a number sequence of the form;

$$a, \quad a + d, \quad a + 2d, \quad a + 3d, \quad \dots, \quad a + (n - 1)d$$

where a is the initial term
d is the common difference
n is number of terms

Notice that the last term, L, is given by

$$L = a + (n - 1)d \qquad n \geqslant 1$$

$$\begin{array}{rccccccc} S = & a & + a + d & + a + 2d & + \dots & + L - 2d & + L - d & + L \\ S = & L & + L - d & + L - 2d & + \dots & + a + 2d & + a + d & + a \\ \hline 2S = & a + L & + a + L & + a + L & + \dots & + a + L & + a + L & + a + L \end{array}$$

$$\therefore \quad 2S = n\{a + L\}$$

$$\therefore \quad S = \frac{n}{2}\{a + L\} \qquad n \geqslant 1$$

n times the average of the first and last terms

The formula book result is now obtained by substituting into this the formula for L:

$$S = \frac{n}{2}\{a + a + (n - 1)d\}$$

$$\therefore S = \frac{n}{2}\{2a + (n - 1)d\} \qquad n \geqslant 1$$

2.3 Exercise

Question 1

(i) Write out the first six terms of the arithmetic progression with first term, a, of 11 and common difference, d, of 7

(ii) Use the formula,

$$L = a + (n - 1)\,d \qquad n \geqslant 1$$

to calculate the 400th term.

(iii) Use the formula,

$$S = \frac{n}{2}\{a + L\} \qquad n \geqslant 1$$

to find the sum of the first 400 terms of the Arithmetic progression.

Question 2

C1 Examination question from May 2007, Q4.

A girl saves money over a period of 200 weeks. she saves 5 p in Week 1,
7 p in Week 2, 9 p in Week 3, and so on until Week 200.
Her weekly savings form an arithmetic sequence.

(a) Find the amount she saves in Week 200.

[3 marks]

(b) Calculate her total savings over the complete 200 week period.

[3 marks]

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Question 3

C1 Examination question from May 2013, Q7

A company, which is making 200 mobile phones each week, plans to increase its production.

The number of mobile phones produced is to be increased by 20 each week from 200 in week 1 to 220 in week 2, to 240 in week 3 and so on, until it is producing 600 in week N.

(a) Find the value of N

[2 marks]

The company then plans to continue to make 600 mobile phones each week.

(b) Find the total number of mobile phones that will be made in the first 52 weeks starting from and including week 1.

[5 marks]

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Question 4

C1 Examination question from June 2009, Q5

A 40-year building programme for new houses began in Oldtown in the year 1951 (Year 1) and finished in 1990 (Year 40).

The numbers of houses built each year form an arithmetic sequence with first term a and common difference d.

Given that 2400 new houses were built in 1960 and 600 new houses were built in 1990, find

(a) the value of d

[3 marks]

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(b) the value of a

[2 marks]

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(c) the total number of houses built in Oldtown over the 40-year period

[3 marks]

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Question 5

Without using a calculator, show how to determine the value of,

$$\sum_{1}^{21} (7n + 3)$$

Question 6

C1 Examination question from January 2009, Q9

The first term of an arithmetic series is a and the common difference is d.

The 18th term of the series is 25 and the 21st term of the series is 32½.

(a) Use this information to write down two equations for a and d.

[2 marks]

(b) Show that $a = -17.5$ and find the value of d.

[2 marks]

The sum of the first n terms of the series is 2750.

(c) Show that n is given by

$$n^2 - 15n = 55 \times 40 \qquad n \geqslant 1$$

[4 marks]

(d) Hence find the value of n.

[3 marks]

Question 7

C1 Examination question from January 2005, Q5

The r^{th} term of an arithmetic series is $(2r - 5)$.

(a) Write down the first three terms of this series.

[2 marks]

(b) State the value of the common difference.

[1 mark]

(c) Show that

$$\sum_{r=1}^{n} (2r - 5) = n(n - 4)$$

[3 marks]