Lesson 7

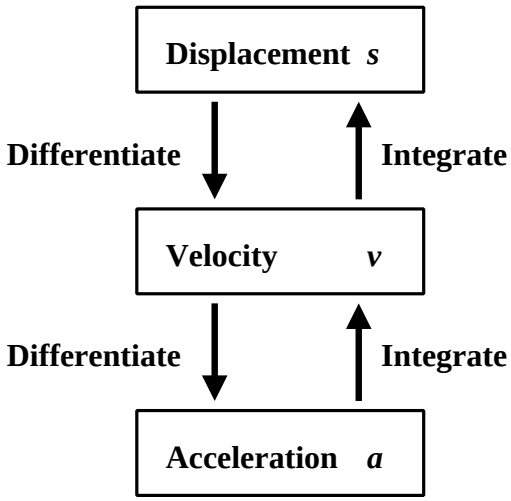
A-Level Pure Mathematics, Year 1
Additional Mathematics
Integration I

7.1 Calculus Kinematics

The calculus of Differentiation and Integration has a straightforward application in Kinematics, the study of motion. It allows the skilled user to hop between the quantities of displacement, *s*, velocity, *v*, and acceleration, *a*.

$$v = \frac{ds}{dt} \qquad a = \frac{dv}{dt}$$

$$s = \int v\,dt \qquad v = \int a\,dt$$



7.2 Example

A particle moves in a straight line.
Its velocity, *v* m s^{-1}, *t* seconds after passing a point *O* is given by the equation

$$v = 8 + \frac{3t^2}{4}$$

(i) Show that the particle's acceleration when $t = 5$ seconds is 7.5 m s^{-2}

[3 marks]

(ii) Show that the distance travelled between the times $t = 1$ and $t = 4$ is 39.75 m

[4 marks]

7.3 Exercise

Any solution based entirely on graphical
or numerical methods is not acceptable
Marks Available : 80

Question 1

Additional Mathematics Examination Question from June 2007, Q2 (OCR)

A particle moves in a straight line.
Its velocity, v m s^{-1}, t seconds after passing a point O is given by the equation

$$v = 6 + 3t^2$$

Find the distance travelled between the times $t = 1$ and $t = 3$

[4 marks]

Question 2

Additional Mathematics Examination Question from June 2008, Q6 (OCR)

A speedboat accelerates from rest so that t seconds after starting its velocity, in m s^{-1}, is given by the formula

$$v = 0.36t^2 - 0.024t^3$$

(i) Find the acceleration at time t

[3 marks]

(ii) Find the distance travelled in the first 10 seconds.

[4 marks]

Question 3

Additional Mathematics Examination Question from June 2010, Q8 (OCR)

A train moves between two stations, taking 5 minutes for the journey.
The velocity of the train may be modelled by the equation

$$v = 60\,(t^4 - 10t^3 + 25t^2)$$

where v is measured in metres per minute and t is measured in minutes.

Calculate the distance between the two stations.

[5 marks]

Question 4

Additional Mathematics Examination Question from June 2005, Q8 (OCR)

A car moves in a straight line.
Its velocity in metres per second, t seconds after passing a point A, is given by

$$v = 27 - \frac{1}{8}t^3$$

It comes to rest at a point B.

(i) Show that the car is at B when $t = 6$

[1 mark]

(ii) Find the distance AB

[5 marks]

Question 5

Additional Mathematics Examination Question from June 2009, Q9 (OCR)

A car accelerates from rest.

At time t seconds, its acceleration is given (in metres per second squared) by

$$a = 4 - 0.2t$$

until $t = 20$

(i) Find the velocity after 5 seconds

[3 marks]

(ii) What is happening to the velocity at $t = 20$?

[1 mark]

(iii) Find the distance travelled in the first 20 seconds

[3 marks]

Question 6

GCSE Examination Question from May 2004, Q13 (OCR)

A bod is moving in a straight line which passes through a fixed point O.

The displacement, s metres, of the body from O at time t seconds is given by

$$s = t^3 + 4t^2 - 5t$$

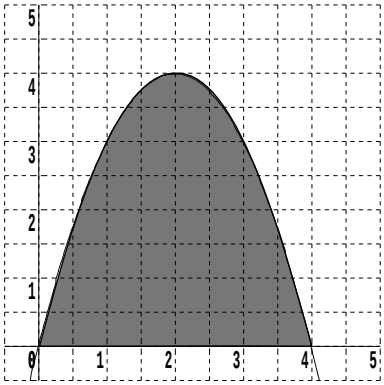
(a) Find an expression for the velocity, v m s^{-1}, at time t seconds

[2 marks]

(b) Find the acceleration after 2 seconds

[2 marks]

Question 7



A cyclist is waiting at a red traffic light.
When the light turns green, he travels along a straight road before stopping at a second red traffic light.
The velocity-time graph shows the motion of the cyclist between the two sets of lights.
The units of velocity are metres per second, those of time are seconds, and those of distance are metres.
The curve has equation

$$v = t\,(4 - t)$$

(i) Use calculus to determine the shaded area.

[3 marks]

(ii) What does the shaded area tell you about the cyclist's journey ?

[1 mark]

(iii) Determine the gradient of the curve at the point (1, 3)

[2 marks]

(iv) What does the gradient at (1, 3) tell you about the cyclist's journey ?

[1 mark]

Question 8

Additional Mathematics Specimen Examination Question from 2002, Q12 (OCR)

A body is falling through a liquid, and the distance fallen is modelled by the formula

$$s = 48t - t^3$$

until it comes to rest.

In the model; s centimetres is the distance fallen
t seconds is the time

(a) Find

(i) the velocity when $t = 1$

[2 marks]

(ii) the initial velocity

[1 mark]

(iii) the acceleration when $t = 1$

[3 marks]

(iv) the time when the body comes to rest

[2 marks]

(v) the distance fallen when the body comes to rest

[2 marks]

(b) Sketch the velocity-time graph for the period of time until the body comes to rest.

[2 marks]

Question 9

Additional Mathematics Examination Question from June 2013, Q12 (OCR)

An object sinks through a thick liquid such that at time t seconds after being released on the surface the depth, s metres is given by

$$s = 4t^2 - \frac{2t^3}{3} \qquad for\ 0 \leqslant t \leqslant 4$$

(a) Find the formula for the velocity, v metres per second, t seconds after being released. hence show that the object stops sinking when $t = 4$.

[4 marks]

(b) Find

(i) the acceleration of the object when it's released on the surface of the liquid

[4 marks]

(ii) the greatest depth of the object

[2 marks]

(c) Sketch the velocity-time & the acceleration-time graphs

[2 marks]

Question 10

Additional Mathematics Examination Question from June 2004, Q13 (OCR)

I regularly travel a journey of 200 kilometres.
When I travel by day I average v kilometres per hour. When I travel at night the traffic is not so bad, so I can average 20 kilometres per hour faster. This means that I am able to complete the journey in 50 minutes less time.

(i) Write down expressions for the journey times during day and at night.

[2 marks]

(ii) Hence form an equation in v and show that it simplifies to

$$v^2 + 20v - 4800 = 0$$

[5 marks]

(iii) Hence find the times it takes me to complete the journey during the day and at night.

[5 marks]

Question 11

GCSE Examination Question from November 2009, Q23 (Edexcel)

In a race, Paula runs 25 laps of a track.

Each lap of the track is 400 m, correct to the nearest metre.

Paula's average speed is 5.0 m s^{-1}, correct to one decimal place.

Calculate the upper bound for the time that Paula takes to run the race.

Give your answer in minutes and seconds, correct to the nearest second.

[4 marks]