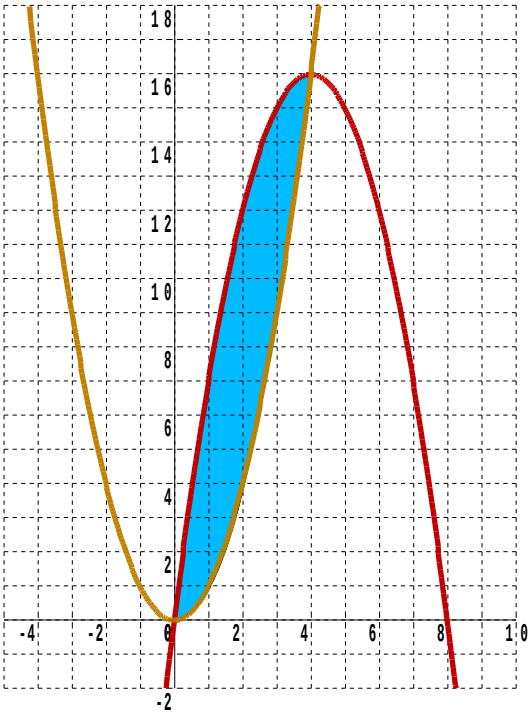
Lesson 4

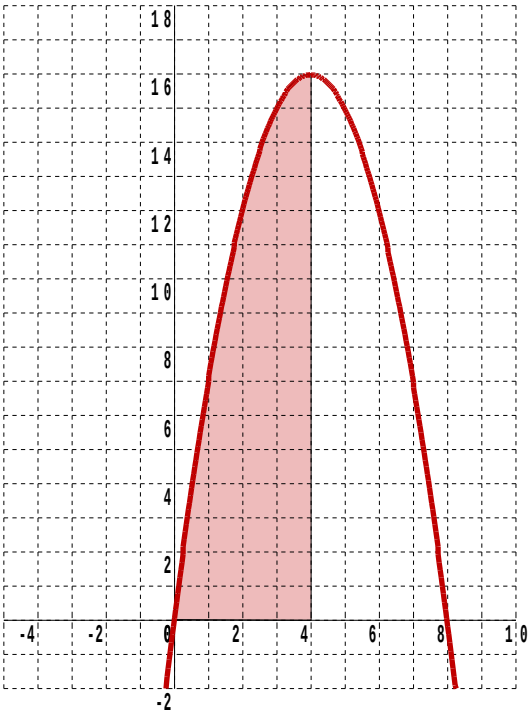
A-Level Pure Mathematics, Year 1
Additional Mathematics
Integration I

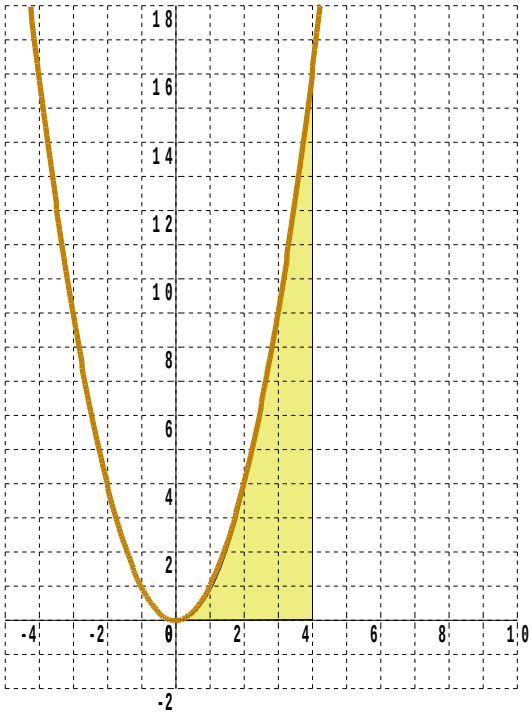
4.1 Areas Between Curves

It is desired to find the area, shaded blue, between two curves.



The strategy is to use integration to find the pink area under the upper red curve and subtract the yellow area under the lower gold curve.





The two curves have the equations,

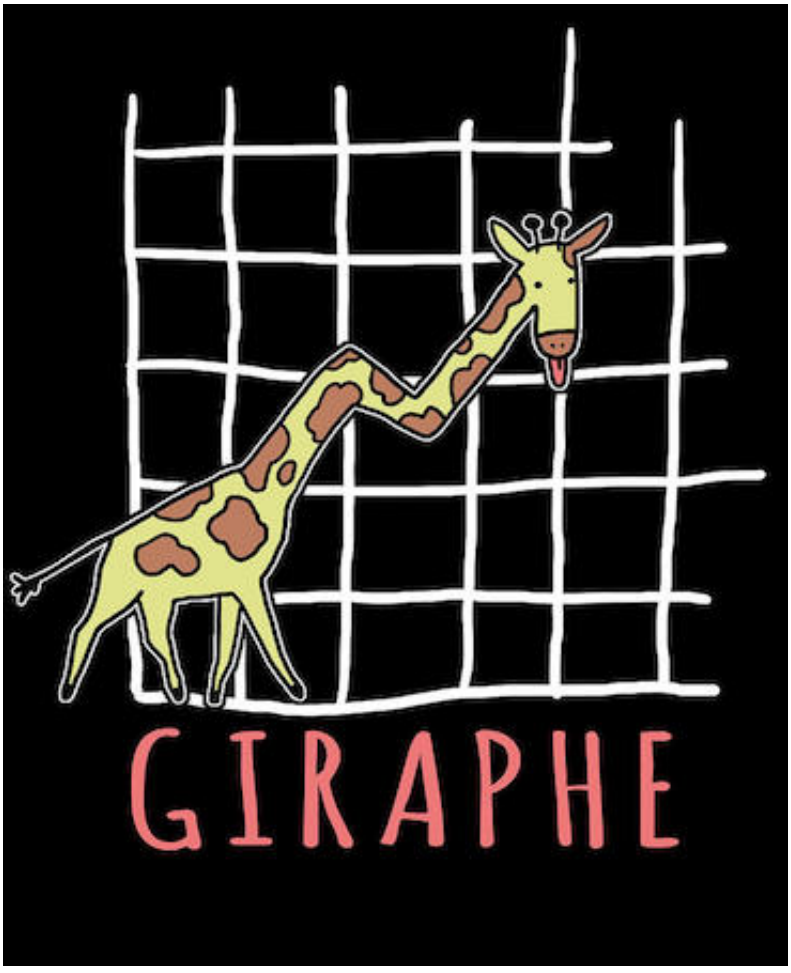
$y = 8x - x^2$ The red 'upside down' parabola

$y = x^2$ The gold parabola

Find area between the two curves.

Teaching Video : http://www.NumberWonder.co.uk/v9043/4.mp4

[6 marks]



4.2 Exercise

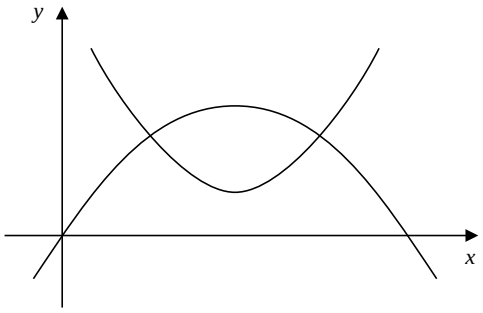
Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available : 52

Question 1

Additional Mathematics Examination Question from June 2007, Q8 (OCR) edited

The figure shows the graphs of $y = 4x - x^2$ and $y = x^2 - 4x + 6$



(i) Use an algebraic method to find the x-coordinates of the points where the curves intersect.

[3 marks]

(ii) Show that the area enclosed by the curves is $\int_1^3 8x - 2x^2 - 6\,dx$

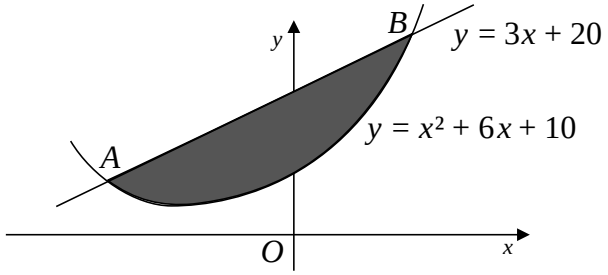
[2 marks]

(iii) Calculate the area enclosed by the two curves.

[4 marks]

Question 2

A-Level Examination Question from January 2005, Paper C2, Q8 (Edexcel)



The line with equation $y = 3x + 20$ cut the curve with equation $y = x^2 + 6x + 10$ at the points *A* and *B*, as shown.

(a) Use algebra to find the coordinates of *A* and the coordinates of *B*.

[5 marks]

The shaded region is bounded by the line and the curve, as shown.

(b) Use calculus to find the exact area shown shaded.

[7 marks]

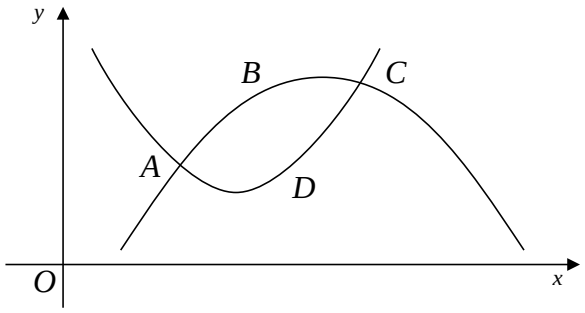
Question 3

Additional Mathematics Examination Question from June 2009, Q11 (OCR)

The shape *ABCD* below represents a leaf.

The curve *ABC* has equation $y = -x^2 + 8x - 9$

The curve *ADC* has equation $y = x^2 - 6x + 11$



(i) Find algebraically the coordinates of *A* and *C*, the points where the curves intersect.

[5 marks]

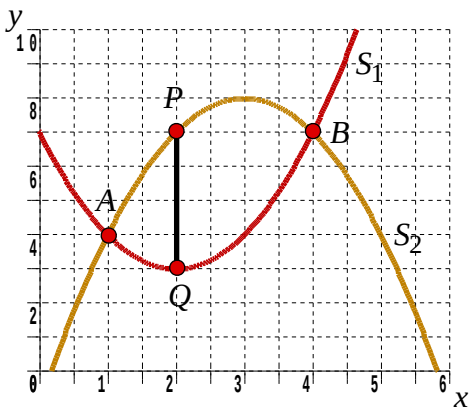
(ii) Find the area of the leaf.

[7 marks]

Question 4

Additional Mathematics Examination Question from June 2015, Q11 (OCR)

Two curves, S_1 and S_2 have equations $y = x^2 - 4x + 7$ and $y = 6x - x^2 - 1$ respectively. The curves meet at A and B.



(i) Show that the coordinates of A and B are $(1, 4)$ and $(4, 7)$ respectively.

[2 marks]

Points P and Q lie on S_2 and S_1 between A and B.

P and Q have the same x coordinate so that PQ is parallel to the y-axis, as shown.

(ii) Find an expression, in its simplest form, for the length PQ as a function of x.

[2 marks]

(iii) Use calculus to find the greatest length of PQ

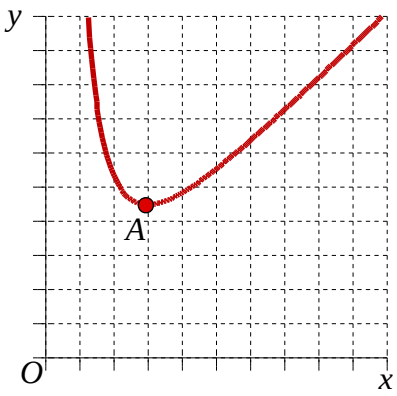
[4 marks]

(iv) Find the area between the two curves.

[4 marks]

Question 5

Additional Mathematics Examination Question, June 2018, Q11 (Cambridge IGCSE)



The diagram shows part of the graph of $y = 16x + \frac{27}{x^2}$ which has a minimum at A

(i) Find the coordinates of A

[4 marks]

The points P and Q lie of the curve $y = 16x + \frac{27}{x^2}$ and have x-coordinates 1 and 3 respectively.

(ii) Find the area enclosed by the curve and the line PQ.
You must show all your working.

[6 marks]

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