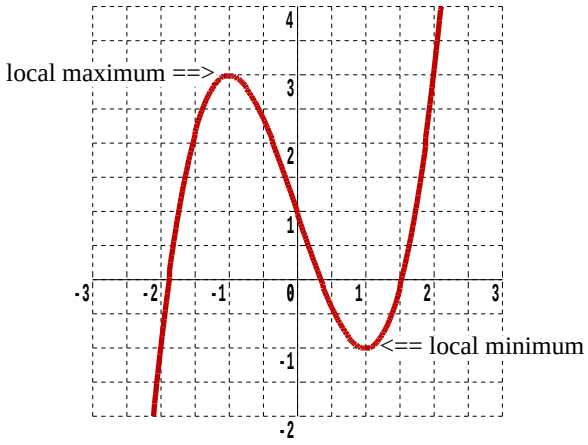
Lesson 6

GCSE
Differentiation I

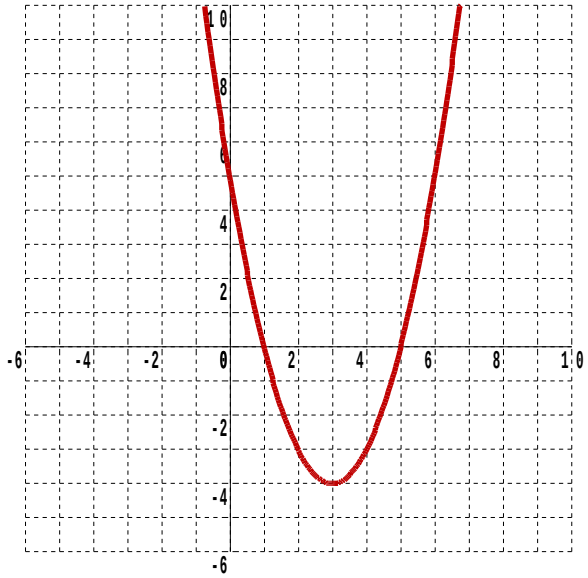
6.1 Local Minimum & Local Maximum

Differentiation is used to find the optimal solutions to problems.
On a graph, such 'best' solutions are often found where there is either a **local maximum** or a **local minimum**.



6.2 The Graphical Method

The graph below is of the equation $y = x^2 - 6x + 5$



By looking at the graph, write down the integer coordinates of the local minimum.

☞

[1 mark]

As mathematicians, we don't want to have to go to the bother of plotting the graph to find this important point.

6.3 The Mathematical Method

Teaching Video : http://www.NumberWonder.co.uk/v9036/6.mp4

The teaching video will talk you through the following method of finding all local minima and local maxima of a function.

Finding Local Minima and Local Maxima

STEP 1 : Differentiate the **POINTS** equation to get its **GRADIENT** equation.
STEP 2 : Set the **GRADIENT** equation equal to zero and solve.
STEP 3 : Put the solution(s) from STEP 2 back into the **POINTS** equation.

Example

Find all local maxima and minima (if any) on the curve with equation;

$$y = x^2 - 6x + 5$$

[3 marks]

6.4 Exercise

Marks Available : 50

Question 1

Find the coordinates of the **local minimum** point on the following quadratic curve;

$$y = x^2 - 8x + 9$$

[3 marks]

Question 2
Find the coordinates of the **local maximum** point on the following parabola,

$$y = 6x + 14 - x^2$$

[3 marks]

Question 3
Find the coordinates of the **local minimum** point on the following parabola,

$$y = 2x^2 - 20x + 52$$

[3 marks]

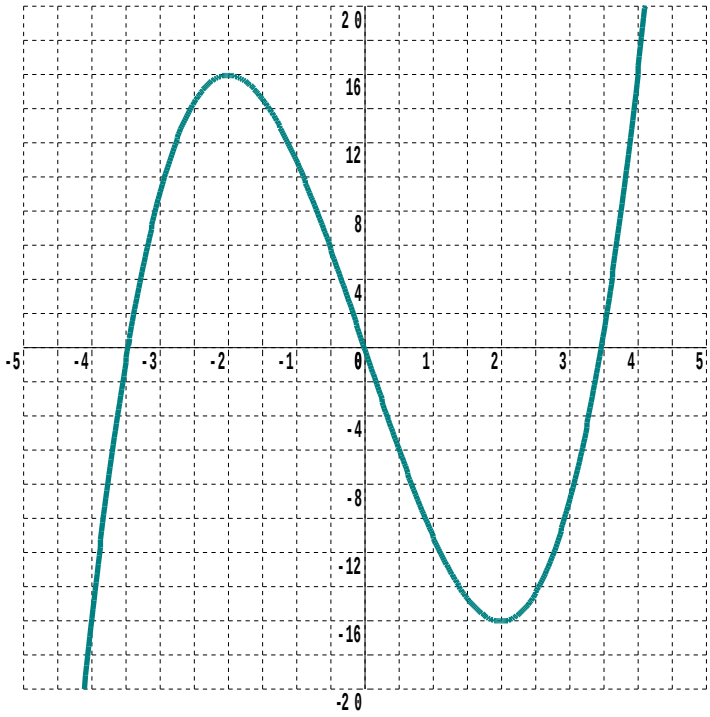
Question 4
Find the coordinates of the **local maximum** point on the following parabola

$$y = 12x - 7 - 3x^2$$

[3 marks]

Question 5

Consider the equation, $y = x^3 - 12x$



(a) From looking at the curve,

(i) write down the coordinates of the **local maximum** point.

[1 mark]

(ii) write down the coordinates of the **local minimum** point.

[1 mark]

(b) Use the mathematical method to obtain the same answers.

[4 marks]

Question 6

Find the coordinates of any **local minimum** or **local maximum** point on;

(i) $y = x^3 - 27x$

[4 marks]

(ii) $y = (x + 7)(x + 1)$

[4 marks]

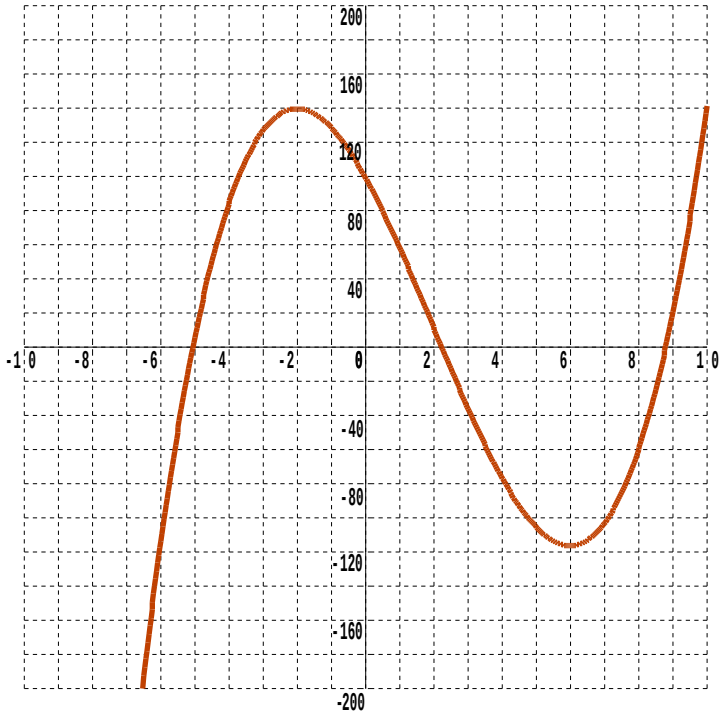
(iii) $y = x^4 - 256x$

[4 marks]

Question 7

Use mathematics to find the local minimum and local maximum of the curve,

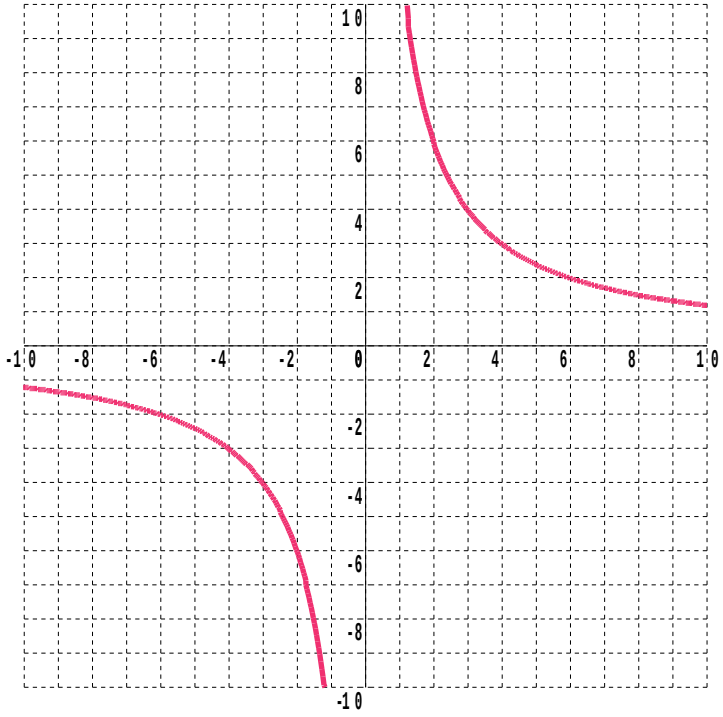
$$y = x^3 - 6x^2 - 36x + 100$$



[6 marks]

Question 8

The graph is of the "inverse proportion" function $f(x) = \frac{12}{x}$



(i) Write down the gradient function, $f'(x)$

[2 marks]

(ii) Write down the bend detector function, $f''(x)$

[2 marks]

(iii) Use the appropriate function to find the point on this curve where $x = 2$

[2 marks]

(iv) Use the appropriate function to find the gradient of this curve when $x = 2$

[2 marks]

(v) Determine if the curve is bending anticlockwise or clockwise when $x = 2$

[2 marks]

Question 9

The curve $y = x^3 + 12x$ has no turning points

Show that this is the case by trying to find them via the mathematical method.
What goes “wrong” ?

[4 marks]