Lesson 4

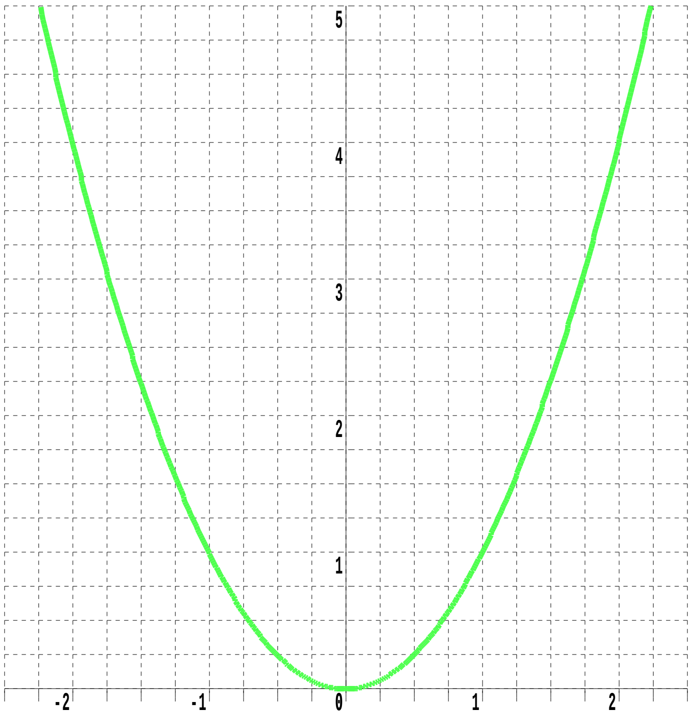
GCSE
Differentiation I

4.1 Point, Gradient and Bend

The graph shown below is of the function $g(x) = x^2$

Previously, the main interest would have focussed on the POINTS on the curve and, indeed, the curve itself may have been plotted by working out points that are on the curve and joining them up, dot-to-dot fashion.

Clearly, $g(x) = x^2$, is best though of as being the POINTS function.



The gradient of the left side of the curve is negative; it slopes downward as the eye starts to moves from left to right. There is then a turning point at (0, 0) where the gradient turns through zero before sloping upwards with positive gradient as the eye continues moving left to right.

Although the gradient is changing, it is not changing randomly. It changes according to the GRADIENT function which is obtained by differentiation.

$$g(x) = x^2 \Rightarrow g'(x) = 2x$$

When $x = 2.5$, for example the gradient is $2 \times 2.5 = 5$

In other words, I can get the value of the gradient at a point $(x, g(x))$ on the curve by putting the x part of the coordinate into the GRADIENT function, $g'(x)$.

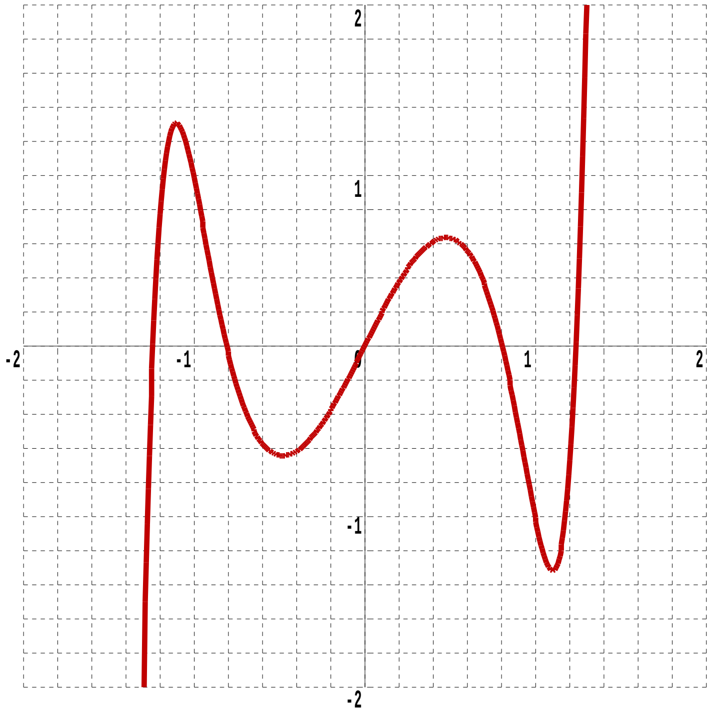
Amazingly, by differentiating a second time, a BEND function is obtained. On putting a value of x into this whether the curve is bending anticlockwise (positive answer) or clockwise (negative answer) is detected;

$$g(x) = x^2 \Rightarrow g'(x) = 2x \Rightarrow g''(x) = 2 \text{ (Positive)}$$

In this case the BEND function is always 2 (Positive) which means the graph always has an anticlockwise curve to it, as can be seen from the graph.

4.2 Example

The graph is of the function, $k(x) = 2x^9 - 3x^7 + x^5 - 3x^3 + 2x$



Teaching Video : http://www.NumberWonder.co.uk/v9036/4a.mp4 (Part 1)
http://www.NumberWonder.co.uk/v9036/4b.mp4 (Part 2)

<= Part 1

Part 2 =>

The video will guide you through the following questions,

(i) Write down the gradient function, $k'(x)$

(ii) Write down the "bend detector" function, $k''(x)$

(iii) Use the appropriate function to find the point on the graph where $x = 1$

(iv) Use the appropriate function to find the gradient of the curve when $x = 1$

(v) Determine if the curve is bending anticlockwise or clockwise when $x = 1$

[10 marks]

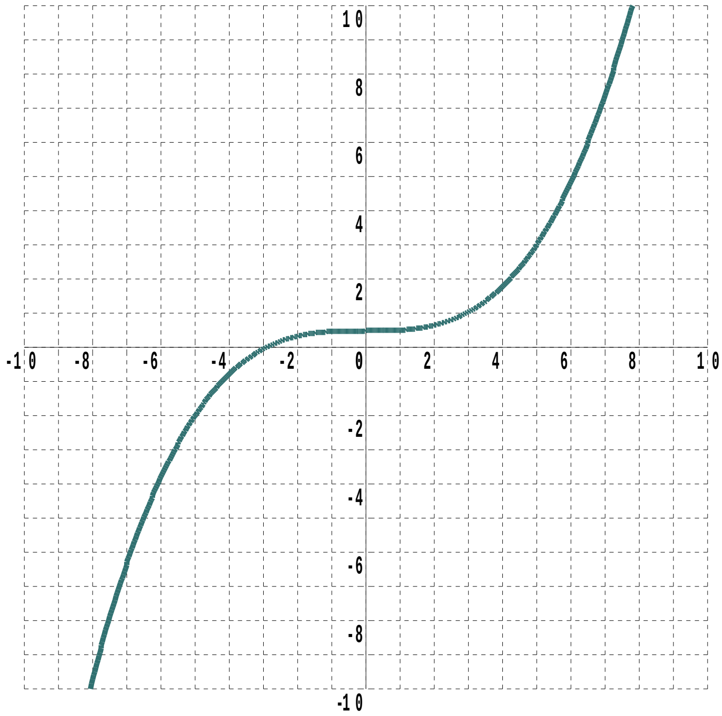
4.3 Exercise

Marks Available : 100

Question 1

The graph is of the function,

$$p(x) = 0.02x^3 + 0.5$$



(i) Write down the gradient function, $p'(x)$

[2 marks]

(ii) Write down the "bend detector" function, $p''(x)$

[2 marks]

(iii) Use the appropriate function to find the point on the graph where $x = 5$

[2 marks]

(iv) Use the appropriate function to find the gradient of the curve when $x = 5$

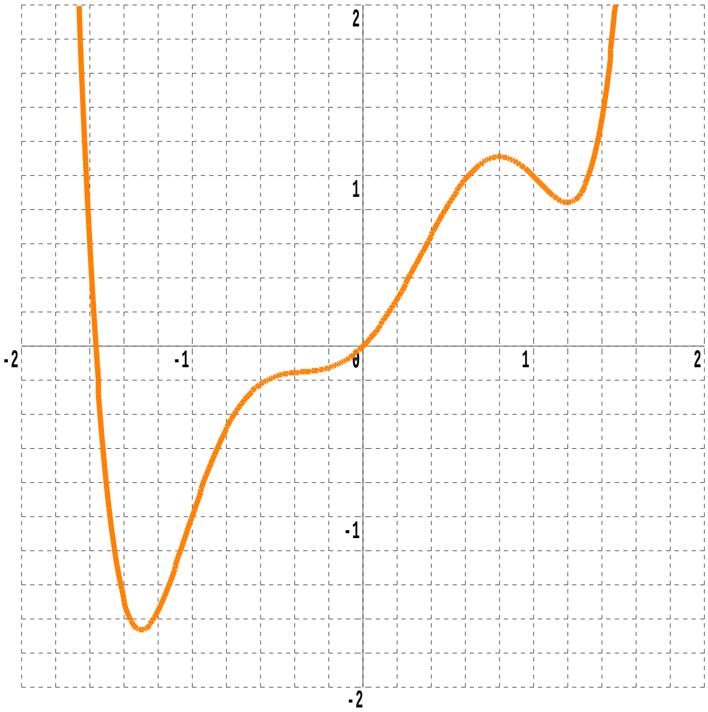
[2 marks]

(v) Determine if the curve is bending anticlockwise or clockwise when $x = 5$

[2 marks]

Question 2

The graph below is of the function $f(x) = x^6 - 3x^4 + 2x^2 + x$



This curve is to be analysed mathematically, with the graph provided so that the mathematical answers can be seen to be correct, or at least plausible.

Think of $f(x) = x^6 - 3x^4 + 2x^2 + x$ as the POINTS function

(a) Work out

(i) $f(0)$ and indicate where the point $(0, f(0))$ is on the graph

[2 marks]

(ii) $f(1)$ and indicate where the point $(1, f(1))$ is on the graph

[2 marks]

(iii) $f(-1)$ and indicate where the point $(-1, f(-1))$ is on the graph

[2 marks]

(b) Differentiate, $f(x) = x^6 - 3x^4 + 2x^2 + x$ to obtain the GRADIENT function of the curve, $f'(x)$

[2 marks]

(c) Work out,

(i) $f'(0)$ and check that the gradient has the up, flat or down slope at the point $(0,\ f(0))$ on the graph that your calculation predicts

[2 marks]

(ii) $f'(1)$ and check that the gradient has the up. flat or down slope at the point $(1,\ f(1))$ on the graph that your calculation predicts

[2 marks]

(iii) $f'(-1)$ and check that the gradient has the up, flat or down slope at the point $(-1,\ f(-1))$ on the graph that your calculation predicts

[2 marks]

(d) Differentiate, $f(x) = x^6 - 3x^4 + 2x^2 + x$ a second time to obtain the BEND function of the curve, $f''(x)$

[2 marks]

(e) Work out,

(i) $f''(0)$ and check on the graph that the bend is curving at the point $(0,\ f(0))$ as your calculation predicts

[2 marks]

(ii) $f''(1)$ and hence state if the graph is bending anticlockwise or clockwise at $(1,\ f(1))$ as it's hard to tell from the graph !

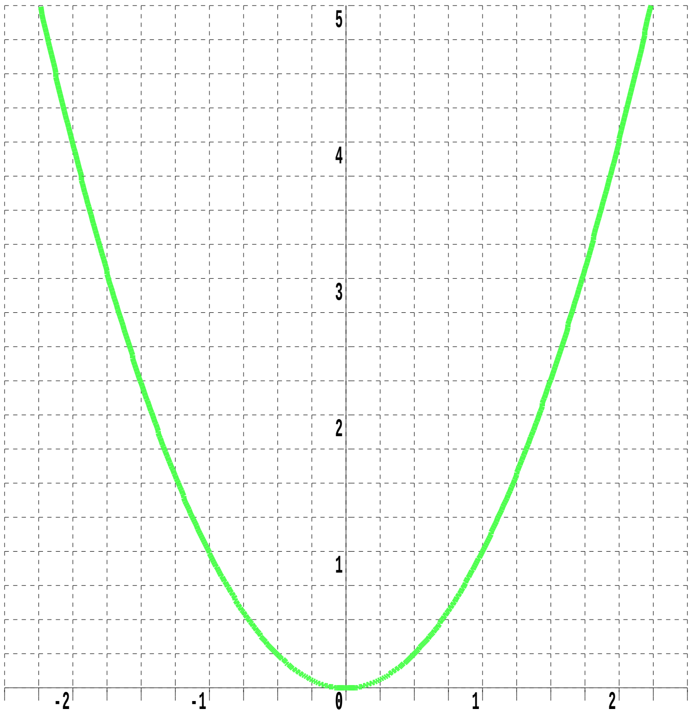
[2 marks]

(iii) $f''(-1)$ and hence state if the graph is bending anticlockwise or clockwise at $(-1,\ f(-1))$ as it's hard to tell from the graph !

[2 marks]

Question 3

The graph of the function $g(x) = x^2$ is given below (again),



(a) Work out

(i) $g(-2)$ and indicate where the point $(-2,\ g(-2))$ is on the graph

[2 marks]

(ii) $g\left(\frac{1}{2}\right)$ and indicate where $\left(\frac{1}{2},\ g\left(\frac{1}{2}\right)\right)$ is on the graph

[2 marks]

(iii) $g\left(\frac{3}{2}\right)$ and indicate where $\left(\frac{3}{2},\ g\left(\frac{3}{2}\right)\right)$ is on the graph

[2 marks]

(b) Differentiate, $g(x) = x^2$ to obtain the GRADIENT function of the curve $g'(x)$

[2 marks]

(c) Work out,

(i) $g'(-2)$ and check that the gradient has the up, flat or down slope at the point $(-2,\ g(-2))$ on the graph that your calculation predicts

[2 marks]

(ii) $g'\left(\frac{1}{2}\right)$ and check that the gradient has the up, flat or down slope at the point $\left(\frac{1}{2},\ g\left(\frac{1}{2}\right)\right)$ on the graph that your calculation predicts

[2 marks]

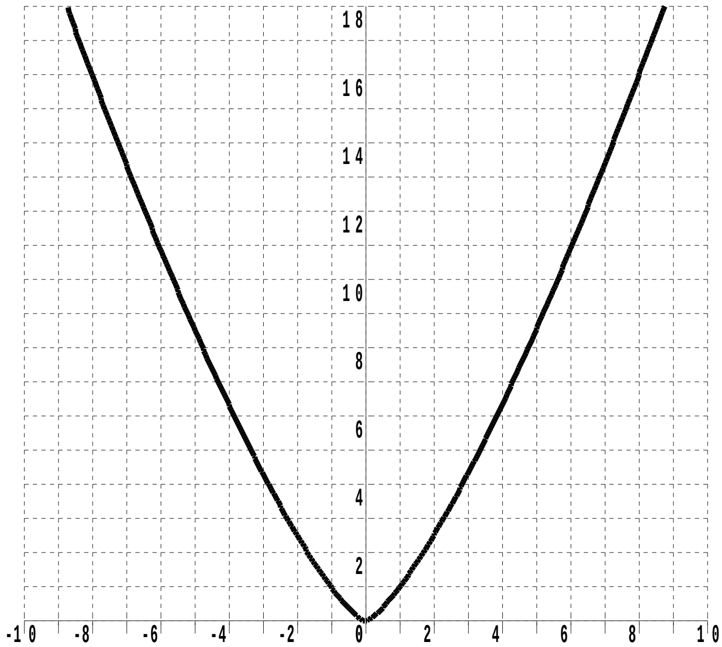
(iii) $g'\left(\frac{3}{2}\right)$ and check that the gradient has the up, flat or down slope at the point $\left(\frac{3}{2},\ g\left(\frac{3}{2}\right)\right)$ on the graph that your calculation predicts

[2 marks]

Question 4

The graph shown below is of the function,

$$h(x) = x^{\frac{4}{3}}$$



(a) Work out

(i) $h(0)$ and indicate where the point $(0, \ h(0))$ is on the graph

[2 marks]

(ii) $h(1)$ and indicate where the point $(1, \ h(1))$ is on the graph

[2 marks]

(iii) $h(8)$ and indicate where the point $(8, \ h(8))$ is on the graph

[2 marks]

(b) Differentiate, $h(x) = x^{\frac{4}{3}}$ to obtain the GRADIENT function of the curve, $h'(x)$

[2 marks]

(c) Work out,

(i) $h'(0)$ and check that the gradient has the down, flat or up slope at the point $(0, h(0))$ on the graph that your calculation predicts

[2 marks]

(ii) $h'(1)$ and check that the gradient has the down, flat or up slope at the point $(1, h(1))$ on the graph that your calculation predicts

[2 marks]

(iii) $h'(8)$ and check that the gradient has the down, flat or up slope at the point $(8, h(8))$ on the graph that your calculation predicts

[2 marks]

(d) Differentiate, $h(x) = x^{\frac{4}{3}}$ a second time to obtain the BEND function of the curve, $h''(x)$

[2 marks]

(e) **(i)** Show that $h''(0) = 0$. This result is a #FAIL of the method. It does not tell you what the bend is doing at $(0,\ h(0))$.

[2 marks]

(ii) Work out $h''(1)$ and check on the graph that the bend is curving at the point $(1,\ h(1))$ as your calculations predict

[2 marks]

(iii) Work out $h''(8)$ and check on the graph that the bend is curving at the point $(8,\ h(8))$ as your calculations predict

[2 marks]

(f) By looking at the graph, or otherwise, try to find the value of x at which this curve has a 45° slope. i.e. Where is the gradient equal to 1 ?

[3 marks]

Question 5

Differentiate the following;

(i) $y = 7x^3 + 4x^2$

(ii) $y = 13x$

(iii) $y = 2.5x^{10}$

(iv) $y = 19$

(v) $y = \dfrac{5}{2}x^4$

(vi) $y = \dfrac{7}{3}x^9$

(vii) $y = 1.5x^3$

(viii) $y = 3.2x$

(ix) $y = 5x^4 + 2x^3 + 7$

(x) $y = 4x^5 + 7x + 1.2$

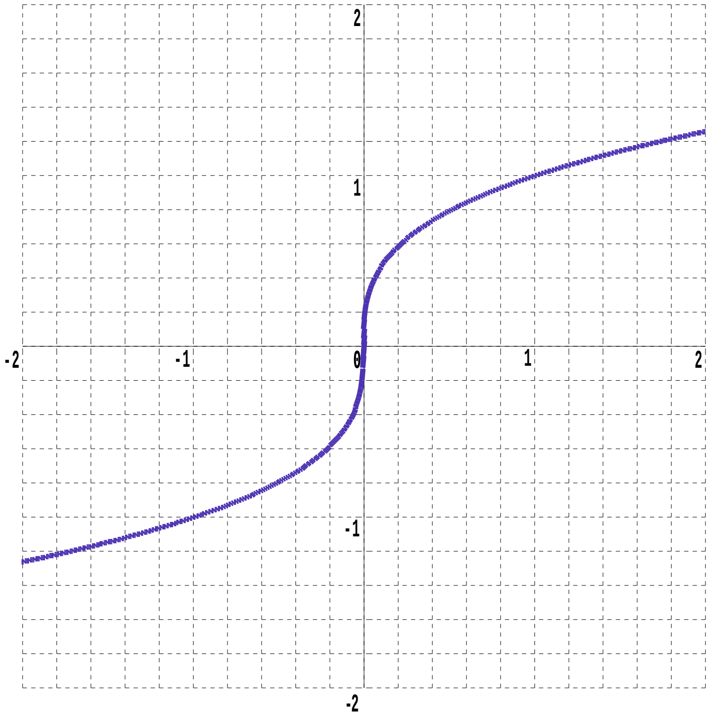
(xi) $y = (4x + 3)(4x + 3)$

(xii) $y = (3x + 5)^2$

[12 marks]

Question 6 * Hard Question

The graph is of the function $m(x) = x^{\frac{1}{3}}$



(a) Work out,

(i) $m\left(\frac{1}{8}\right)$ and indicate where the point $\left(\frac{1}{8},\ m\left(\frac{1}{8}\right)\right)$ is on the graph.

[2 marks]

(ii) $m(1)$ and indicate where the point $(1,\ m(1))$ is on the graph

[2 marks]

(iii) $m\left(\frac{216}{125}\right)$ and indicate where the point $\left(\frac{216}{125},\ m\left(\frac{216}{125}\right)\right)$ is on the graph

[2 marks]

(b) Differentiate, $m(x) = x^{\frac{1}{3}}$ to obtain the GRADIENT function of the curve, $m'(x)$

[2 marks]

(c) Work out,

(i) $m'\left(\frac{1}{8}\right)$ and check that the gradient has the down, flat or up slope at the point $\left(\frac{1}{8},\ m\left(\frac{1}{8}\right)\right)$ on the graph that your calculation predicts

[2 marks]

(ii) $m'(1)$ and check that the gradient has the down, flat or up slope at the point $(1,\ m(1))$ on the graph that your calculation predicts

[2 marks]

(iii) $m'\left(\frac{216}{125}\right)$ and check that the gradient has the down, flat or up slope at the point $\left(\frac{216}{125},\ m\left(\frac{216}{125}\right)\right)$ that your calculation predicts

[2 marks]

(d) By looking at the graph, or otherwise, try to find the value of x at which this curve has a $45°$ slope. i.e. Where is the gradient equal to 1 ?

[3 marks]