

## Lesson 7

**A-Level Pure Mathematics, Year 1**  
**Additional Mathematics**  
**Coordinate Geometry**

### 7.1 Circles in Disguise

Consider the equation,  $x^2 + y^2 + 12x - 2y = 27$

A friend of mine claims that this is a circle. If they are correct then it must be possible to algebraically manipulate this equation into the form

$$(x - a)^2 + (y - b)^2 = r^2$$

where  $a$ ,  $b$  and  $r$  are constants the values of which need to be found.

Then, the circle's centre would be  $(a, b)$  and its radius  $r$ .

### 7.2 Completing the Square

The technique employed is called “completing the square”.

Teaching Video : <http://www.NumberWonder.co.uk/v9033/7.mp4>



### 7.3 Exercise

*Any solution based entirely on graphical or numerical methods is not acceptable*

Marks Available : 70

#### Question 1

Write each of the following in the “completed square” form,

$$y = (x + a)^2 + b$$

(i)  $y = x^2 + 8x + 17$

(ii)  $y = x^2 + 10x + 7$

(iii)  $y = x^2 - 12x + 3$

(iv)  $y = x^2 - 6x - 7$

[ 8 marks ]

#### Question 2

Consider the circle,  $x^2 - 4x + y^2 - 8y = 44$

(i) Rewrite this in the form

$$(x - a)^2 + (y - b)^2 = r^2$$

where  $a$ ,  $b$  and  $r$  are constants the values of which are to be found.

[ 4 marks ]

(ii) Hence, or otherwise, state;

(a) The coordinates of the centre of the circle

[ 1 mark ]

(b) The radius of the circle.

[ 1 mark ]

**Question 3**

Expand the brackets and simplify;

**(i)**  $y = (x + 3)^2 + 4$

**(ii)**  $y = (x + 7)^2 - 10$

**(iii)**  $y = (x + 1)^2 + 7$

**(iv)**  $y = \left(x + \frac{1}{2}\right)^2 + 10$

**[ 8 marks ]****Question 4**Consider the circle,  $x^2 + y^2 + 8x - 14y + 29 = 0$ **(i)** Rewrite this in the form

$$(x - a)^2 + (y - b)^2 = r^2$$

where  $a$ ,  $b$  and  $r$  are constants the values of which are to be found.**[ 4 marks ]****(ii)** Hence, or otherwise, state;**(a)** The coordinates of the centre of the circle**[ 1 mark ]****(b)** The radius of the circle**[ 1 mark ]**

**Question 5**

*Additional Mathematics Examination Question from June 2007, Q3 (OCR)*

A circle has equation  $x^2 + y^2 - 4x - 6y + 3 = 0$

Find the coordinates of the centre and the radius of the circle

[ 4 marks ]

**Question 6**

*Additional Mathematics Examination Question from June 2015, Q9 (OCR)*

The equation of a circle is  $x^2 + y^2 - 8x + 2y - 19 = 0$

( i ) Express the equation of  $C$  in the form  $(x - a)^2 + (y - b)^2 = r^2$

[ 4 marks ]

( ii ) Hence or otherwise, use an algebraic method to decide whether the point  $( 8, 3 )$  lies inside, outside or on the circumference of the circle. Show all your working.

[ 2 marks ]

**Question 7**

*Additional Mathematics Examination Question from June 2010, Q9 (OCR)*

The diameter of a circle is  $PQ$ , where  $P(1, 3)$  and  $Q(15, 1)$

- (i) Find the centre of the circle

[ 2 marks ]

- (ii) Show that the radius of the circle is  $5\sqrt{2}$

[ 2 marks ]

- (iii) Hence find the equation of the circle in the form

$$x^2 + y^2 + ax + by + c = 0$$

[ 2 marks ]

**Question 8**

Consider the circle  $x^2 + y^2 + 41 = 10(x + y)$

- (i) Rewrite this in the form

$$(x - a)^2 + (y - b)^2 = r^2$$

where  $a$ ,  $b$  and  $r$  are constants the values of which are to be found.

[ 4 marks ]

- (ii) Hence, or otherwise, state;

- (a) The coordinates of the centre of the circle

[ 1 mark ]

- (b) The radius of the circle.

[ 1 mark ]

**Question 9**

*Additional Mathematics Examination Question from June 2005, Q12 (OCR)*

- ( i ) A circle has equation  $x^2 + y^2 - 2x - 4y - 20 = 0$   
Find the coordinates of its centre,  $C$ , and its radius.

[ 3 marks ]

- ( ii ) Find the coordinates of the points,  $A$  and  $B$ , where the line  
 $y = x + 2$  cuts the circle

[ 5 marks ]

- ( iii ) Find angle  $ACB$

[ 4 marks ]

**Question 10**

*A-Level Examination Question from May 2011, Paper C2, Q4 (Edexcel)*

The circle  $C$  has equation

$$x^2 + y^2 + 4x - 2y - 11 = 0$$

Find

( a ) the coordinates of the centre of  $C$

[ 2 marks ]

( b ) the radius of  $C$

[ 2 marks ]

( c ) the coordinates of the points where  $C$  crosses the  $y$ -axis,  
giving your answers as simplified surds.

[ 4 marks ]