

2.1 Line Segments

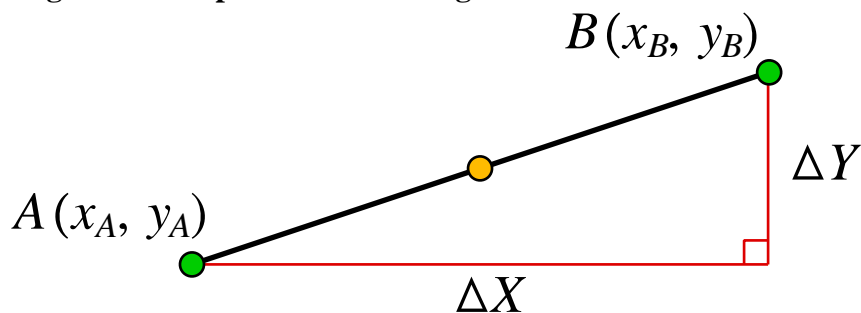
A line segment is simply a piece of straight line between two endpoints.

Finding the length of a line segment is straight forward using Pythagoras' Theorem.

The coordinates of the midpoint of a line segments are found by taking the average (the mean) of the x parts of the two endpoints, to give the x part of the midpoint, and, separately, taking the average (the mean) of the y parts of the two endpoints to give the y part of the endpoint.

These comments are formalised in the following theorem;

The Length and Midpoint of a Line Segment



A line segment with endpoints $A(x_A, y_A)$ and $B(x_B, y_B)$ has a length given by;

$$|AB| = \sqrt{(\Delta X)^2 + (\Delta Y)^2}$$

where $\Delta X = x_B - x_A$ and $\Delta Y = y_B - y_A$

Furthermore,

$$\text{Midpoint } AB = \left(\frac{x_B + x_A}{2}, \frac{y_B + y_A}{2} \right)$$

The ΔX and ΔY are the same as that used to calculate gradient, $m = \frac{\Delta Y}{\Delta X}$

In three dimensions with endpoints $A(x_A, y_A, z_A)$ and $B(x_B, y_B, z_B)$ the theorem is only marginally more complicated;

$$|AB| = \sqrt{(\Delta X)^2 + (\Delta Y)^2 + (\Delta Z)^2}$$

where $\Delta X = x_B - x_A$, $\Delta Y = y_B - y_A$ and $\Delta Z = z_B - z_A$

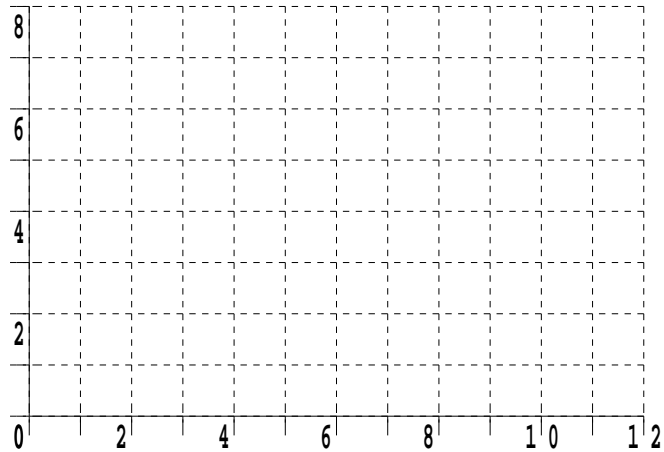
$$\text{Midpoint } AB = \left(\frac{x_B + x_A}{2}, \frac{y_B + y_A}{2}, \frac{z_B + z_A}{2} \right)$$

2.2 Example

The graph of this function is a line segment.

$$f(x) = \frac{1}{2}x + 1, \quad x \in \mathbb{R}, \quad 2 \leq x \leq 10$$

Sketch the line segment on the grid below, and determine its length and midpoint using the theorem, “The Length and Midpoint of a Line Segment”



Teaching Video : <http://www.NumberWonder.co.uk/v9033/2.mp4>



[7 marks]

2.3 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

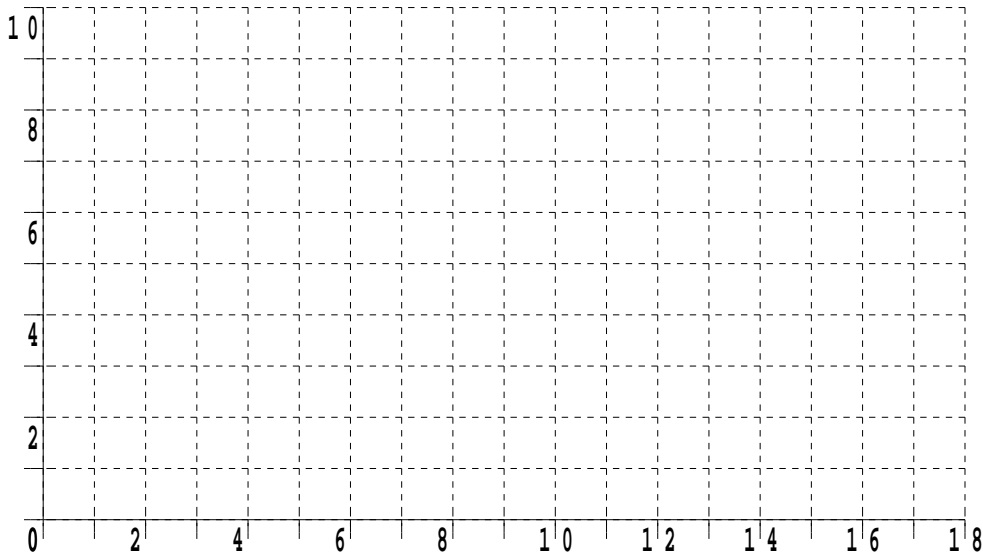
Marks Available : 55

Question 1

The graph of this function is a line segment.

$$g(x) = -\frac{1}{3}x + 8, \quad x \in \mathbb{R}, \quad 3 \leq x \leq 15$$

- (i) Sketch the line segment on the grid below



[3 marks]

- (ii) Use the theorem “The Length and Midpoint of a Line Segment” to calculate the exact length of the line segment.

[2 marks]

- (iii) Use the theorem “The Length and Midpoint of a Line Segment” to determine midpoint of the line segment.

[2 marks]

Question 2

Without drawing a graph, and showing your working, determine the exact distance between the two points $A(-15, -2)$ and $B(9, 5)$

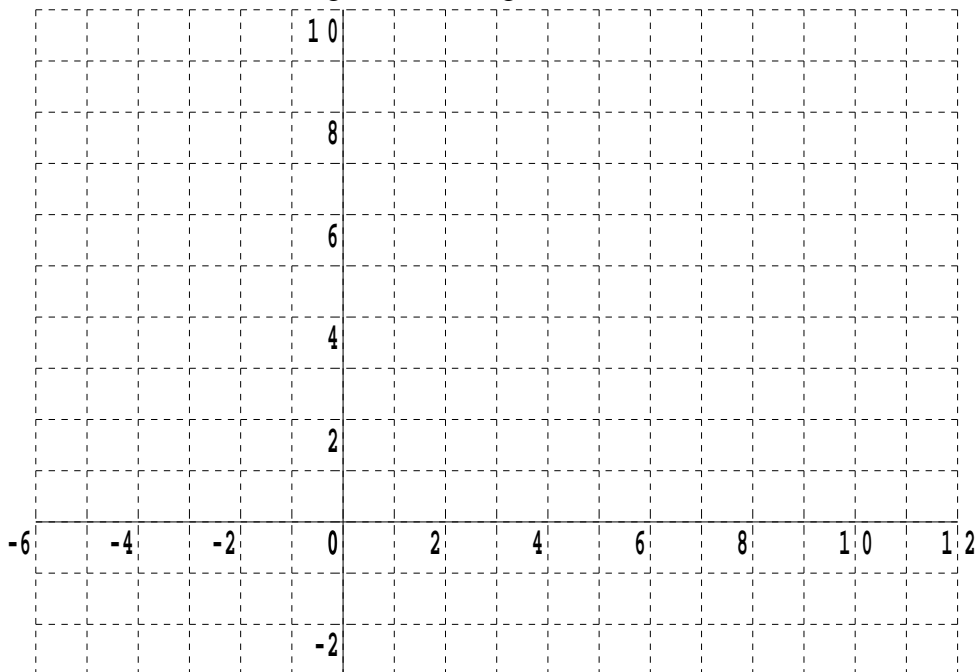
[3 marks]

Question 3

The graph of this function is a line segment.

$$h(x) = \frac{2}{3}x + 1, \quad x \in \mathbb{R}, \quad -3 \leq x \leq 9$$

- (i) Sketch the line segment on the grid below



[3 marks]

- (ii) Use the theorem “The Length and Midpoint of a Line Segment” to calculate the exact length of the line segment.

[2 marks]

- (iii) Use the theorem “The Length and Midpoint of a Line Segment” to determine midpoint of the line segment.

[2 marks]

Question 4

A straight line has equation $4y - 12x + 3 = 0$

Write this equation in the form $y = mx + c$

[2 marks]

Question 5

IGCSE Examination Question from January 2020, Paper 1H, Q1 (Edexcel)

The point A has coordinates $(5, -4)$

The point B has coordinates $(13, 1)$

(a) Work out the coordinates of the midpoint of AB

[2 marks]

Line L has equation $y = 2 - 3x$

(b) Write down the gradient of line L

[1 mark]

(c) Does the point with coordinates $(100, -302)$ lie on L ?
You must give a reason for your answer

[1 mark]

Question 6

IGCSE Examination Question from January 2018, Paper 3H, Q10 (Edexcel)

The straight line L is parallel to the line with equation $2y + 8x = 5$

L passes through the point with coordinates $(2, 3)$

Find an equation for L

[3 marks]

Question 7

Additional Mathematics Examination Question from June 2017, Q4 (OCR, FSMQ)

The coordinates of A and B are $(1, 5)$ and $(-3, 7)$ respectively

(i) Calculate the exact length of AB

[2 marks]

(ii) Find the coordinates of the midpoint of AB

[1 mark]

Question 8

Additional Mathematics Examination Question from June 2009, Q3 (OCR, FSMQ)

A is the point $(1, 5)$ and C is the point $(3, p)$

(i) Find the equation of the line through A which is parallel to $2x + 5y = 7$

[2 marks]

(ii) This line also passes through the point C .
Find the value of p

[2 marks]

Question 9

IGCSE Examination Question from June 2017, Paper 4H, Q13 (Edexcel)

Here are the equations of four straight lines,

Line **A** $y = 2x + 3$

Line **B** $2y = 6 - 3x$

Line **C** $4x - 2y = 3$

Line **D** $y = 3 - 2x$

Two of these lines are parallel.

(a) Which two lines ?

[2 marks]

Line **L** has a gradient of $-\frac{5}{2}$ and passes through the point with coordinates (1, 3)

(b) Find an equation of **L**

Give your answer in the form $ax + by = c$ where a , b and c are integers

[3 marks]

Question 10

A-Level Examination Question from January 2008, C1, Q4.

The point $A(-6, 4)$ and the point $B(8, -3)$ lie on the line L .

(a) Find an equation for L in the form $ax + by + c = 0$,
where a , b and c are integers.

[4 marks]

(b) Find the distance AB , giving your answer in the form
 $k\sqrt{5}$, where k is an integer.

[3 marks]

Question 11

A-Level Examination Question from May 2010, C1, Q8

- (a) Find an equation of the line joining $A (7, 4)$ and $B (2, 0)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers

[3 marks]

- (b) Find the length of AB , leaving your answer in surd form

[2 marks]

The point C has coordinates $(2, t)$, where $t > 0$, and $AC = AB$

- (c) Find the value of t

[1 mark]

- (d) Find the area of triangle ABC

[2 marks]