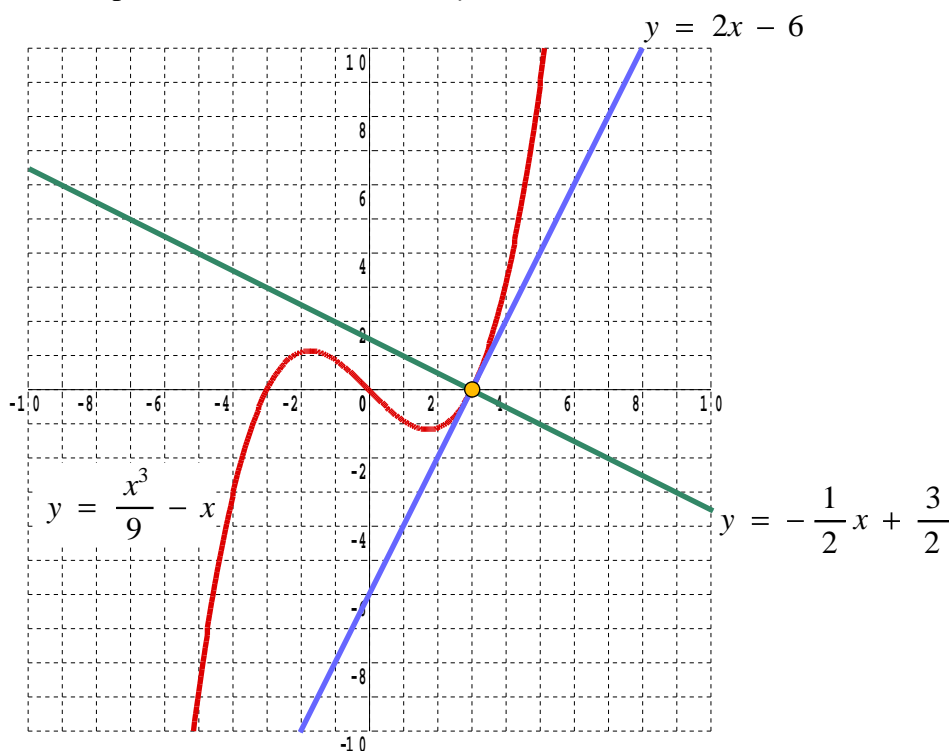


## Lesson 11

### A-Level Pure Mathematics, Year 1 Additional Mathematics Coordinate Geometry

#### 11.1 Normal from Curve

Previously, the curve with equation  $y = \frac{x^3}{9} - x$  was studied and the tangent to it at the point  $(3, 0)$  found to be  $y = 2x - 6$



There is a second line of interest, called the “normal” that is a right angles to the tangent at any specified point. At the point  $(3, 0)$  the normal to the curve

$$y = \frac{x^3}{9} - x \text{ turns out to be } y = -\frac{1}{2}x + \frac{3}{2}.$$

Notice that the gradient of the tangent,  $m_t$ , and the gradient of the normal,  $m_n$  have the property of any pair of mutually perpendicular lines, that,

$$m_t \times m_n = -1$$

In other words, each is the sign changed reciprocal of the other.

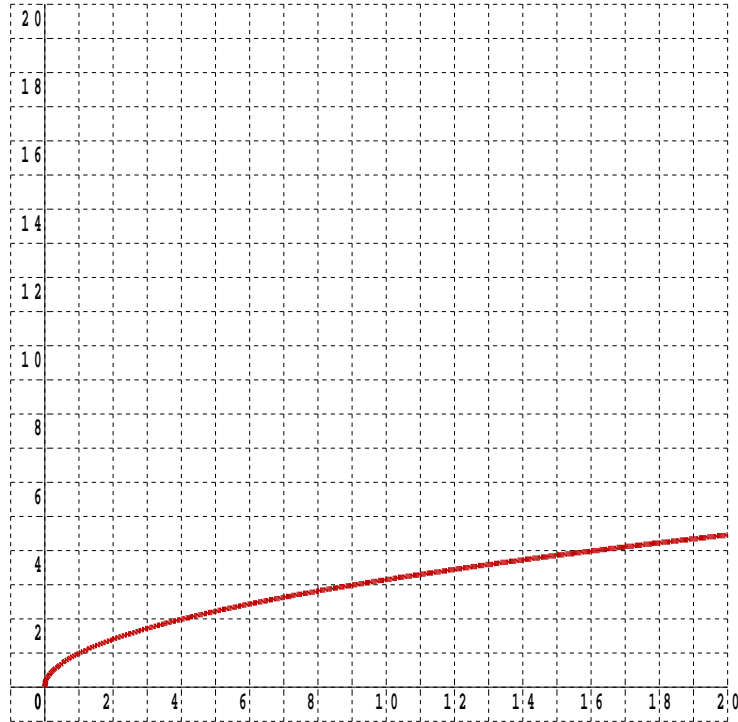
#### 11.2 Why the Normal is of Interest

Imagine the graph to be a road map and the curve a road. A car moves along the road with constant speed. The tangent represents the direction a car moving along the road has at any moment. The normal represents the direction along which the force felt by a person in the car acts as it moves around each bend. It does not give the magnitude of the force which depends on how sharply the road is bending and, indeed, on a straight piece of road the force along the normal has magnitude zero. The force along the normal is often referred to as centripetal force.

### 11.3 Example

The equation of a curve is  $y = \sqrt{x}$

- (i) Find the equation of the normal to this curve at the point where  $x = 4$
- (ii) To the graph below add the part (i) normal



Teaching Video : <http://www.NumberWonder.co.uk/v9033/11.mp4>



[ 5 marks ]

### 11.4 Exercise

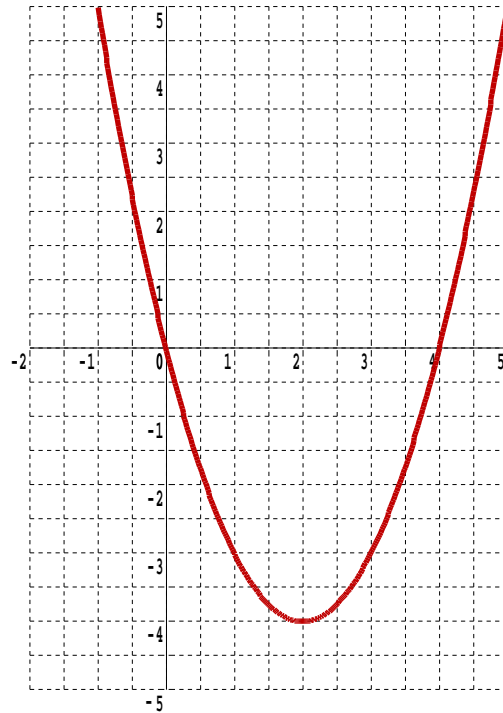
*Any solution based entirely on graphical or numerical methods is not acceptable*

Marks Available : 52

#### Question 1

The equation of a curve is  $y = x^2 - 4x$

- (i) Find the equation of the normal to this curve at the point where  $x = 4$
- (ii) To the graph below add the part (i) normal



[ 5 marks ]

**Question 2**

*Additional Mathematics Examination Question from June 2009, Q2 (OCR)*

Find the equation of the normal to the curve

$$y = x^3 + 5x - 7$$

at the point ( 1, - 1 )

[ 5 marks ]

**Question 3**

*Additional Mathematics Examination Question from June 2019, Paper 1, Q3 (OCR)*

Find the equation of the normal to the curve

$$y = x^3 - 2x^2 + 2x + 4$$

at the point ( 2, 8 )

[ 6 marks ]

**Question 4**

*Additional Mathematics Examination Question from June 2018, Q7 (OCR)*

- (i) Find the coordinates of the points where the line  $y = 7x - 9$  cuts the curve  $y = x^2 + 2x - 5$

[ 4 marks ]

- (ii) Determine whether the line is a normal to the curve at either of the points of intersection

[ 3 marks ]

**Question 5**

*Additional Mathematics Examination Question from June 2014, Q10 (OCR)*

- ( i ) Find the coordinates of the point  $P$  on the curve  $y = 2x^2 + x - 5$   
where the gradient of the curve is 5

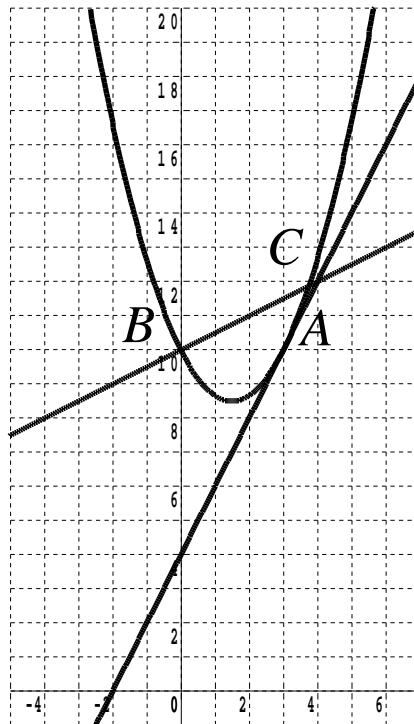
[ 3 marks ]

- ( ii ) Find the equation of the normal to the curve at the point  $P$

[ 3 marks ]

**Question 6**

*Additional Mathematics Examination Question from June 2005, Q10 (OCR)*



The curve shown has equation;

$$y = \frac{2}{3}x^2 - 2x + 10$$

- (i) Find the equation of the tangent to the curve at A ( 3, 10 )

[ 4 marks ]

( ii ) Show that the equation of the normal to the curve at  $B ( 0, 10 )$  is

$$2y - x = 20$$

[ 3 marks ]

( iii ) Find the coordinates of the point  $C$  where these two lines intersect

[ 3 marks ]

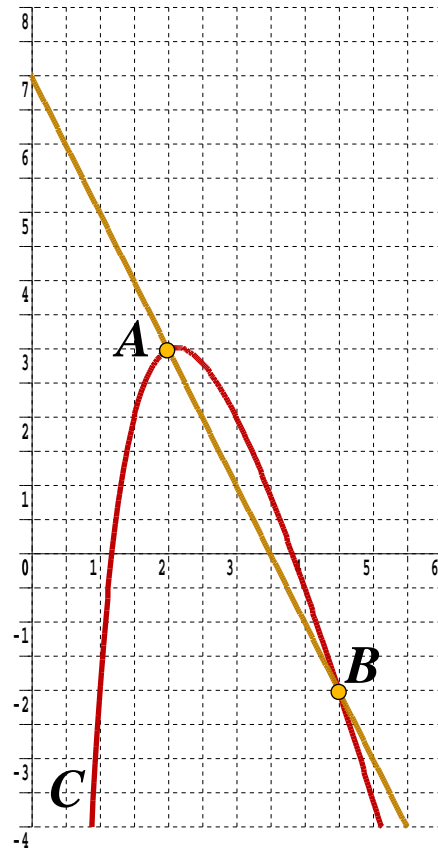
( iv ) Calculate the length  $BC$

[ 2 marks ]



### Question 7

A-Level Examination Question from May 2014, IAL, Paper C1(R), Q11 (Edexcel)



The sketch is of part of the curve  $C$  with equation  $y = 20 - 4x - \frac{18}{x}$ ,  $x > 0$

Point  $A$  lies on  $C$  and has an  $x$  coordinate equal to 2

(a) Show that the equation of the normal to  $C$  at  $A$  is  $y = -2x + 7$

[ 6 marks ]

The normal to  $C$  at  $A$  meets  $C$  again at the point  $B$

(b) Use algebra to find the coordinates of  $B$

[ 5 marks ]