

### 10.1 Tangent from Curve

Previously looked at was "How to find the equation of the tangent to a circle". The mathematics needed to find the tangents to more general curves is straight forward, provided an ability to *differentiate* is already in place.

### 10.2 Differentiation

This is the remarkable ability to take the equation of many curves and, without any working, simply write down the gradient equation of that curve.

#### The Power Rule

$$\text{If } y = x^n \text{ then } \frac{dy}{dx} = n x^{n-1} \text{ for any constant, } n$$

#### Example #1

Write down the derivative of the following curve then check your answer with that given at the foot of the next page.

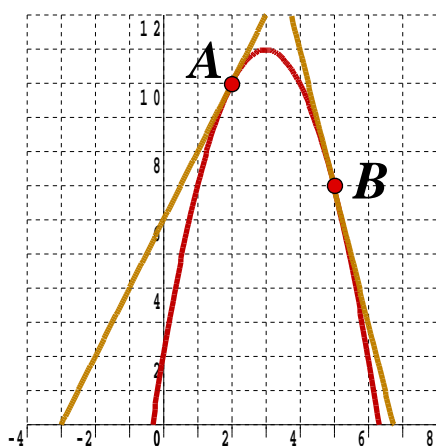
$$y = 7x^4 + 0.5x + 17$$

$$\Rightarrow \frac{dy}{dx} =$$

[ 2 marks ]

### 10.3 The Key Idea

The gradient of the tangent to a point on a curve is the same as the gradient of the the curve at that point.



The red parabola,  $P$ , has equation  $y = 2 + 6x - x^2$  and so  $\frac{dy}{dx} = 6 - 2x$

At  $A(2, 10)$ ,  $P$  has gradient  $2$  and the tangent is  $y = 2x + 6$

At  $B(5, 7)$ ,  $P$  has gradient  $-4$  and the tangent is  $y = -4x + 27$

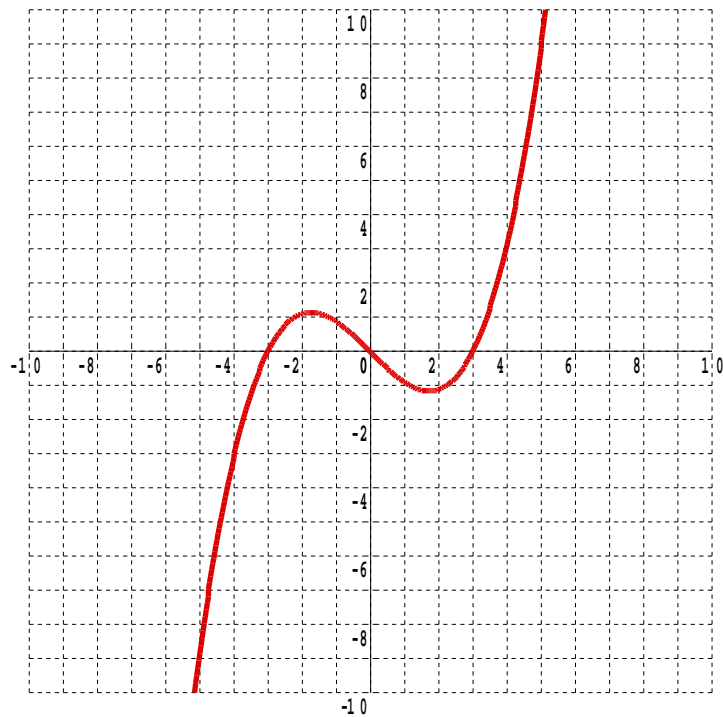
### 10.4 Making Use of The Key Idea

The key idea is often used to get the equation of the tangent to a curve at a given point starting from only the equation of the curve.

#### Example #2

The equation of a curve is  $y = \frac{x^3}{9} - x$

- (i) Find the equation of the tangent to this curve at the point ( 3, 0 )
- (ii) To the graph below add the part (i) tangent



Teaching Video : <http://www.NumberWonder.co.uk/v9033/10.mp4>



[ 6 marks ]

10.5 Example #1 Answer :  $y = 7x^4 + 0.5x + 17 \Rightarrow \frac{dy}{dx} = 28x^3 + 0.5$

## 10.6 Exercise

*Any solution based entirely on graphical or numerical methods is not acceptable*

Marks Available : 50

### Question 1

Differentiate

$$y = 4x^8 - 3x^5$$

[ 2 marks ]

### Question 2

Write down the derivative of

$$y = 15x^{0.4}$$

[ 2 marks ]

### Question 3

Differentiate

$$y = 7 + \frac{1}{2}x^6$$

[ 2 marks ]

### Question 4

Determine  $\frac{dy}{dx}$  of the following expression

$$y = 0.2x^9 + 0.1x$$

[ 2 marks ]

### Question 5

Write down the gradient equation for the following curve

$$y = x^{-5}$$

[ 2 marks ]

### Question 6

By first expanding the brackets, differentiate the following

$$y = (3x + 8)(2x + 1)$$

[ 3 marks ]

**Question 7**

Find the derivative of the following

$$y = \frac{x^3}{9} + 4$$

[ 2 marks ]

**Question 8**

(i) Differentiate;

$$y = \frac{12}{x^2}$$

[ 2 marks ]

**Question 9**

Find the numerical value of the gradient at the point ( 1, 5 ) on the curve

$$y = x^3 + 4$$

[ 3 marks ]

**Question 10**

Find the numerical value of the gradient at the point ( 2, 3.2 ) on the curve,

$$y = 0.1x^5$$

[ 2 marks ]

**Question 11**

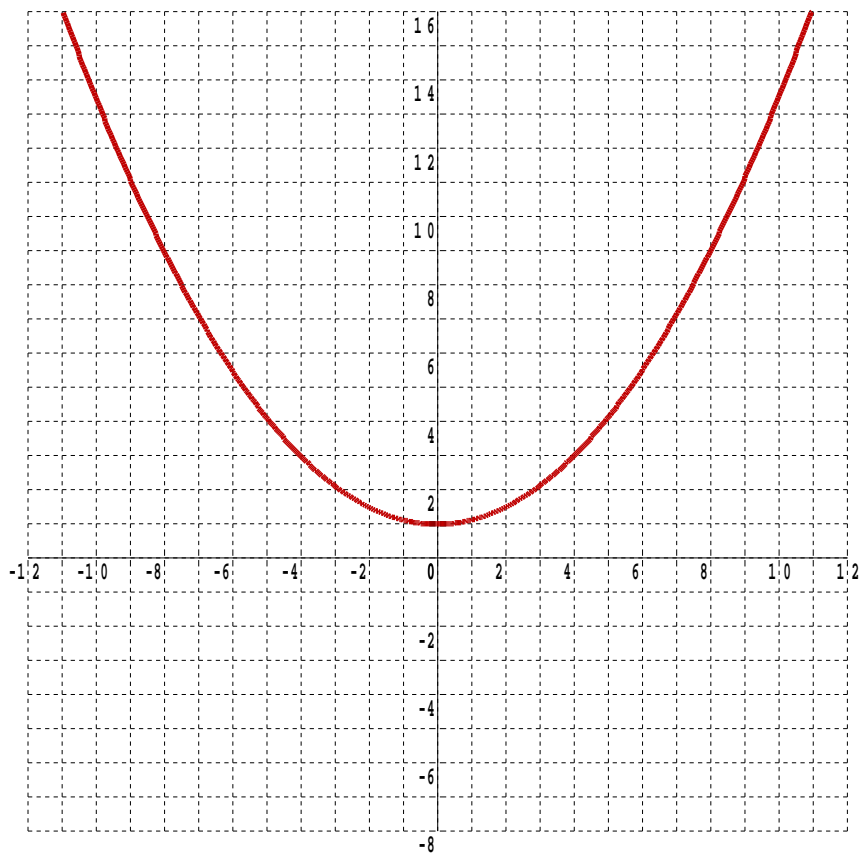
The equation of a curve is

$$y = \frac{x^2}{8} + 1$$

- ( i ) Find the equation of the tangent to this curve at the point ( 8, 9 )

[ 4 marks ]

- ( ii ) To the graph of the curve add your part (i) tangent



[ 2 marks ]

**Question 12**

*Additional Mathematics Examination Question from June 2011, Q2 (OCR)*

The equation of a curve is

$$y = x^3 - x^2 - 2x - 3$$

Find the equation of the tangent to this curve at the point ( 3, 9 )

[ 5 marks ]

**Question 13**

*Additional Mathematics Examination Question from June 2007, Q6 (OCR)*

Find the equation of the tangent to the curve

$$y = x^3 - 3x + 4$$

at the point ( 2, 6 )

[ 4 marks ]

**Question 14**

$$f(x) = \frac{x^3 - x + 8}{16x}, \quad x > 0$$

- (i) Show that  $f(x) = Ax^2 + B + Cx^{-1}$ , where  $A, B$  and  $C$  are constants to be determined.

[ 4 marks ]

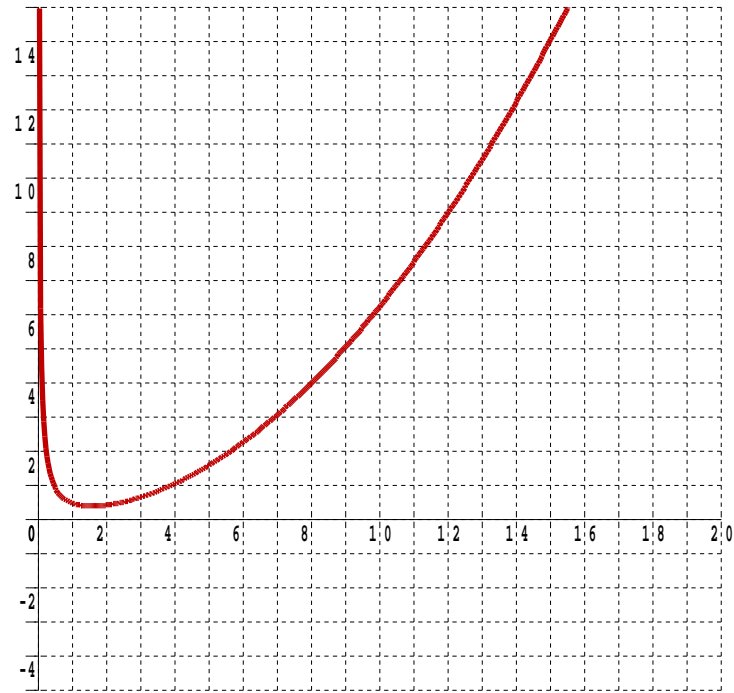
- (ii) Hence find  $f'(x)$

[ 2 marks ]

- (iii) Find an equation of the tangent to the curve  $y = f(x)$  when  $x = 8$

[ 4 marks ]

- (iv) The graph of  $f(x)$  is plotted below.  
Add the tangent calculated in part (iii) to this graph.



[ 3 marks ]