Lesson 3

A-Level Pure Mathematics, Year 1
Additional Mathematics
The Algebra of Polynomials

3.1 Less Long Division

Faced with a cubic equation, an ability to do algebraic long division gave a method of factorising the cubic. The main effort involved was in initially finding one of the factors. Once a first factor was found, then the other two factors were easy to extract from the quotient quadratic. Having to try to find that initial factor is best done, not by algebraic long division, but by making use of The Factor Theorem.

The Factor Theorem

If, for a given polynomial function $p(x)$, $p(a) = 0$ (for some constant, a)

then $(x - a)$ is a factor of $p(x)$

3.2 Example

(a) Use the factor theorem to determine which, if any, of the following are factors of $f(x) = x^3 + 8x^2 + 5x - 50$

(i) $(x + 1)$

(ii) $(x - 2)$

(b) Factorise $f(x)$ fully

(c) Solve the equation $f(x) = 0$

Teaching Video : http://www.NumberWonder.co.uk/v9029/3a.mp4 (Part 1)
http://www.NumberWonder.co.uk/v9029/3b.mp4 (Part 2)

<= Part 1 Part 2 =>

[2, 2, 4, 1 marks]

3.3 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available : 50

Question 1

$$f(x) = 2x^3 + 3x^2 - 3x - 2$$

Use the factor theorem to show that $(x + 2)$ is a factor of $f(x)$

[2 marks]

Question 2

$$g(x) = x^3 + 6x^2 - 19x - 84$$

Use the factor theorem to show that $(x - 3)$ is not a factor of $g(x)$

[2 marks]

Question 3

$$h(x) = x^3 - 3x^2 - 9x + 27$$

(a) **(i)** Determine the value of $h(1)$

[1 mark]

(ii) What does this tell you about $(x - 1)$ with regards to $h(x)$?

[1 mark]

(b) **(i)** Determine the value of $h(3)$

[1 mark]

(ii) What does this tell you about $(x - 3)$ with regards to $h(x)$?

[1 mark]

Question 4

$$k(x) = x^3 - 3x^2 - 36x - 48$$

Here is an algebraic long division of $k(x)$ by $(x + 1)$,

$$\begin{array}{rr|l} & & x^2 - 4x - 32 \\ \hline x + 1 & x^3 - 3x^2 - 36x - 48 & \\ & x^3 + x^2 & \\ \hline & -4x^2 - 36x & \\ & -4x^2 - 4x & \\ \hline & -32x - 48 & \\ & -32x - 32 & \\ \hline & -16 & \end{array}$$

From studying this long division, what do you deduce about $k(x)$ and $(x + 1)$?
Give a reason for your answer

[2 marks]

Question 5

$$m(x) = x^3 + 12x^2 - 4x - 240$$

Use the factor theorem to decide if $(x - 4)$ is a factor of $m(x)$

[3 marks]

Question 6

$$n(x) = x^3 - x^2 - x - 2$$

Use the factor theorem to find a factor of $n(x)$

[3 marks]

Question 7

$$f(x) = x^3 + 5x^2 - 8x - 48$$

(i) Show by algebraic long division that $(x - 3)$ is a factor of $f(x)$

[4 marks]

(ii) Hence factorise $f(x)$ completely

[2 marks]

(iii) Solve the equation, $f(x) = 0$

[1 mark]

Question 8

$$g(x) = x^3 + 6x^2 - 15x - 100$$

(a) Use the factor theorem to determine which, if any, of the following are factors of $g(x)$;

(i) $(x - 4)$

[2 marks]

(ii) $(x + 2)$

[2 marks]

(b) Factorise $g(x)$ fully

[4 marks]

(c) Solve the equation $g(x) = 0$

[1 mark]

Question 9

$$h(x) = x^3 + 18x^2 + 108x + 216$$

(a) Determine

(i) $h(-4)$

[2 marks]

(ii) $h(-6)$

[2 marks]

(b) Factorise $h(x)$ fully

[4 marks]

(c) Solve the equation $h(x) = 0$

[1 mark]

Question 10

$f(x) = x^3 + ax^2 + bx - 12$, for some constants a and b

(i) Given that $(x - 1)$ is a factor of $f(x)$ write down an equation which must be satisfied by a and b

[2 marks]

(ii) Given that $(x + 2)$ is a factor of $f(x)$ write down another equation which must be satisfied by a and b

[2 marks]

(iii) Solve your part (i) and part (ii) equations simultaneously to find the value of a and the value of b

[3 marks]

(iv) Hence, or otherwise, factorise $f(x)$ completely

[2 marks]