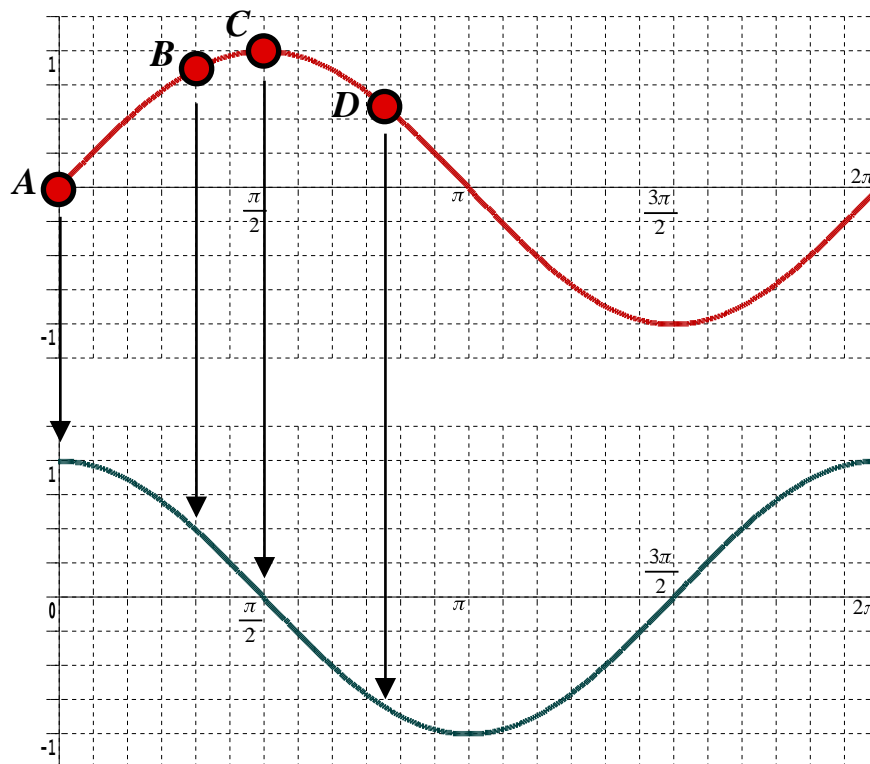


8.1 The Sine Function

The upper graph shows the curve $y = \sin x$ with x measured in radians. Of interest is the gradient of this graph.



Consider what the gradient of the upper graph is doing at the four points labelled.

A : The curve at the origin seems to be sloping like the line $y = x$, gradient 1.

B : The gradient between A and B is positive and reducing in magnitude.

C : At this maximum turning point the gradient has fallen to zero.

D : The gradient between C and D is negative and increasing in magnitude.

Continuing in this manner, and plotting the gradients as a separate graph, the lower graph is obtained. It looks very much like the graph of cosine !

This argument, although not a proof, is an intuitive reasoning of the important result that, provided radians are used,

Derivative of Sine

$$\text{If } y = \sin x \text{ then } \frac{dy}{dx} = \cos x \quad \text{where } x \text{ is in radians}$$

8.2 Differentiating Sine Functions

The Product Rule and The Quotient Rule can be applied to situations where the sine function is involved. So too can The Chain Rule, as follows,

The Chain Rule for $y = \sin(f(x))$

$$\text{If } y = \sin(f(x)) \text{ then } \frac{dy}{dx} = \cos(f(x)) \times f'(x)$$

8.3 Examples

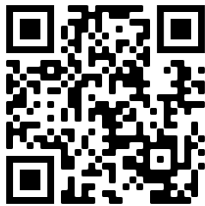
Differentiate each of the following,

(i) $y = \sin^4 3x$ (Chain Rule Example)

(ii) $y = x \sin(x^2)$ (Product Rule Example)

(iii) $y = \frac{\sin(2x)}{e^{2x}}$ (Quotient Rule Example)

Teaching Video : <http://www.NumberWonder.co.uk/v9028/8.mp4>



Watch the video and
then write out the
solutions here



[3, 3, 3 marks]

8.4 Exercise

Marks Available : 40

Question 1

Differentiate each of the following with respect to x ,

(i) $y = 7 \sin(5x)$

(ii) $y = 6 \sin(3x^3)$

[2, 2 marks]

(iii) $y = 2 \sin^3 x$

(iv) $y = \sqrt{\sin x}$

[2, 2 marks]

(v) $y = 11 e^{\sin x}$

(vi) $y = 3 e^{2 \sin(5x)}$

[2, 2 marks]

(vii) $y = \ln(\sin x)$

(viii) $y = \sin(\ln x)$

[2, 2 marks]

Question 2

By writing $y = \csc x$ as $y = (\sin x)^{-1}$ and using The Chain Rule, show that,

Derivative of $\csc x$

If $y = \csc x$ then $\frac{dy}{dx} = -\csc x \cot x$ where x is in radians

[4 marks]

Question 3

$$f(x) = x \sin x$$

(i) Remembering to use radians, find the exact value of $f\left(\frac{\pi}{6}\right)$

[2 marks]

(ii) Use The Product Rule to differentiate $f(x)$

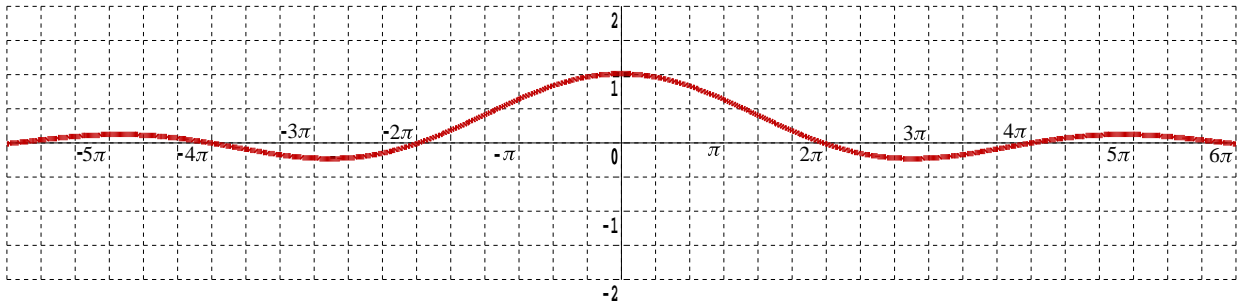
[2 marks]

(iii) Show that $f'\left(\frac{\pi}{6}\right) = \frac{6 + \sqrt{3} \pi}{12}$

[2 marks]

Question 4

The graph is of the important function, $f(x) = \frac{\sin x}{x}$



- (i) The graph is a first hint of the key result that $\lim_{x \rightarrow 0} \frac{\sin x}{x}$
Explain where one should be looking on the graph to suspect this result.

[2 marks]

- (ii) Use The Quotient Rule to differentiate $f(x)$

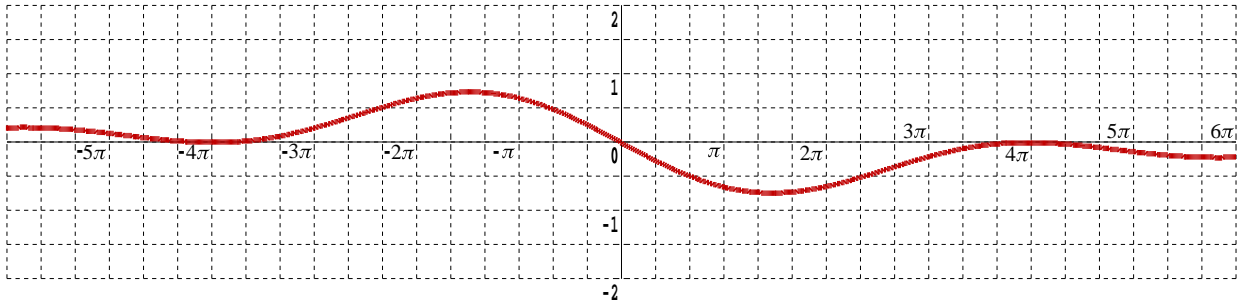
[2 marks]

- (iii) Determine the exact value of $f'(\pi)$

[2 marks]

Question 5

The graph is of another important function, $f(x) = \frac{\cos x - 1}{x}$



Use the graph to deduce a key result for $\lim_{x \rightarrow 0} \frac{\cos x - 1}{x}$

Explain where you were looking on the graph to deduce your result.

[2 marks]

Question 6

Prove from first principles that the derivative of $\sin x$ is $\cos x$

Make use of the limit studied in Question 4 and the limit studied in Question 5 along with the following video from “The Math Sorcerer”

Teaching Video : <http://www.NumberWonder.co.uk/v9028/8b.mp4>



[6 marks]