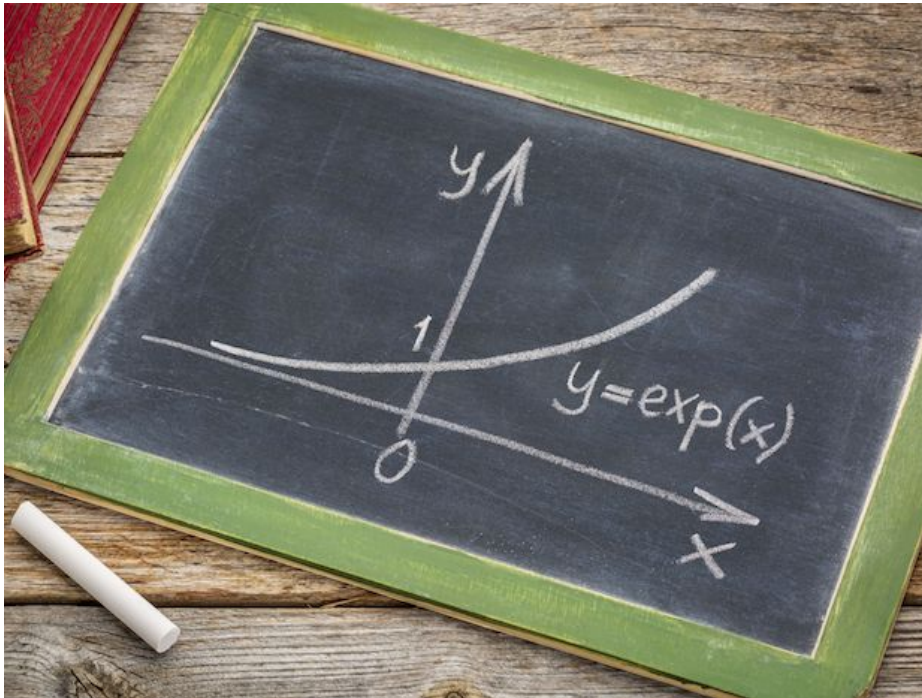


6.1 The Exponential Function

The function $y = e^x$ has the remarkable property of being its own derivative.

The Derivative of $y = e^x$

$$\text{If } y = e^x \text{ then } \frac{dy}{dx} = e^x$$

6.2 The Value of e

e is the number 2.71828 18284 59045 23536 02875 ...

Like π this is an irrational number and like π it crops up in many surprising places throughout mathematics.

6.3 Differentiating Exponentiated Functions

The Product Rule and The Quotient Rule can be applied to situations where the exponential function is involved. So too can The Chain Rule, as follows,

The Chain Rule for $y = e^{f(x)}$

$$\text{If } y = e^{f(x)} \text{ then } \frac{dy}{dx} = e^{f(x)} f'(x)$$

6.4 Examples

Differentiate each of the following,

(i) $y = e^{8x^3 + 5x^2}$ (Chain Rule Example)

(ii) $y = x e^{3x}$ (Product Rule Example)

(iii) $y = \frac{x}{e^x}$ (Quotient Rule Example)

Teaching Video : <http://www.NumberWonder.co.uk/v9028/6.mp4>



Watch the video and
then write out the
solutions here



[3, 3, 3 marks]

6.5 Exercise

Marks Available : 50

Question 1

Differentiate each of the following with respect to x ;

(i) $y = e^{5x}$

(ii) $y = e^{3x^5}$

[2, 2 marks]

(iii) $y = 8e^{6x} + 6e^{5x}$

(iv) $y = 14e^x$

[2, 2 marks]

(v) $y = e^{-7x}$

(vi) $y = \frac{4}{e^{5x}}$

[2, 2 marks]

Question 2

Remembering that when dividing same base indices subtract, differentiate wrt x ,

$$y = \frac{e^{7x}}{e^{3x}}$$

[2 marks]

Question 3

By first expanding the brackets, differentiate with respect to x ,

$$y = e^{3x}(e^{4x} + 7e^{-9x})$$

[3 marks]

Question 4

Use The Product Rule to differentiate each of the following with respect to x .
Simplify to obtain an elegant answers.

(i) $y = x^3 e^x$

(ii) $y = x e^{-x}$

[3, 3 marks]

Question 5

Use The Quotient Rule to differentiate each of the following with respect to x .
Simplify to obtain elegant answers.

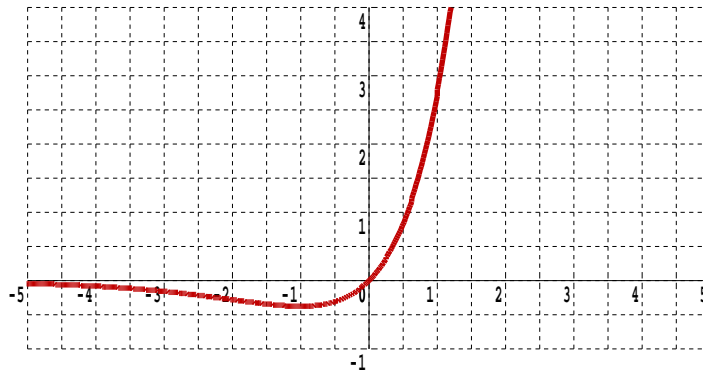
(i) $y = \frac{e^x}{x}$

[3 marks]

(ii) $y = \frac{2 - x^2}{e^{2x}}$

[3 marks]

Question 6



A curve has equation $y = x e^x$

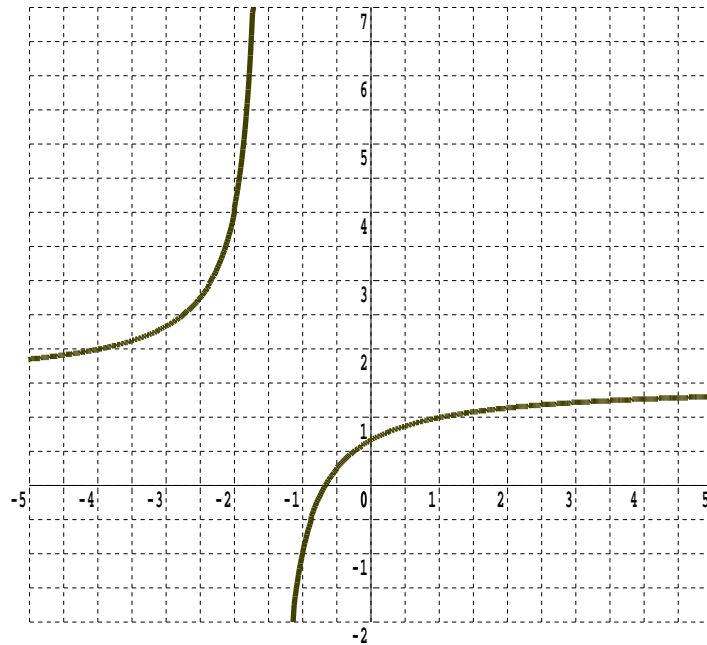
(i) Find $\frac{dy}{dx}$

[3 marks]

(ii) Find the equation of the tangent to the curve when $x = 0$
Draw this tangent onto the graph, above

[4 marks]

Question 7



A curve has equation, $y = \frac{3x + 2}{2x + 3}$

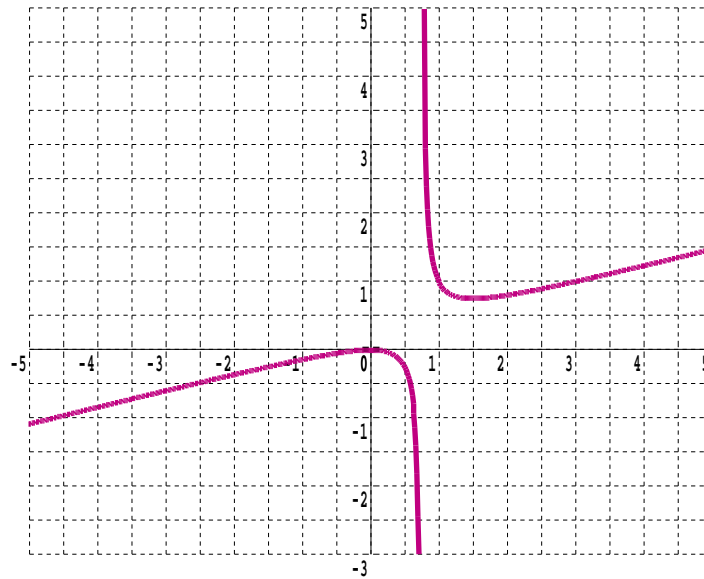
(i) Find $\frac{dy}{dx}$

[3 marks]

(ii) Find the equation of the normal to the curve when $x = 1$
Draw this normal onto the graph, above

[4 marks]

Question 8



A curve has equation $y = \frac{x^2}{(4x - 3)}$

- (i) Find an expression for $\frac{dy}{dx}$

[3 marks]

- (ii) Find the equation of the tangent to the curve when $x = 1$
Draw this tangent onto the graph, above.

[4 marks]