

**5.1 The Quotient Rule**

Given two functions,  $u(x)$  and  $v(x)$ , the first divided by the second, The Quotient Rule gives a method of obtaining the derivative of the division. It states that,

$$\left( \frac{u(x)}{v(x)} \right)' = \frac{v(x) u'(x) - v'(x) u(x)}{(v(x))^2}$$

All of the  $x$  in brackets are considered to be unnecessary clutter and so the rule is more usually written in the following succinct and elegant form,

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**The Quotient Rule**

$$\text{If } f = \frac{u}{v} \text{ then } f' = \frac{v u' - v' u}{v^2}$$

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**5.2 Example**

Differentiate  $y = \frac{x+4}{x+5}$  by immediately applying The Quotient Rule

Teaching Video : <http://www.NumberWonder.co.uk/v9028/5.mp4>



Watch the video and  
then write out the  
solution here



### 5.3 Exercise

Marks Available : 40

#### Question 1

Given that  $y = \frac{4x}{x + 3}$

use The Quotient Rule to show that the derivative is given by  $\frac{dy}{dx} = \frac{12}{(x + 3)^2}$

[ 3 marks ]

#### Question 2

Given that  $y = \frac{x^2}{(x + 5)}$

use The Quotient Rule to show that the derivative is given by  $\frac{dy}{dx} = \frac{x(x + 10)}{(x + 5)^2}$

[ 3 marks ]

**Question 3**

Given that  $y = \frac{5x - 2}{3x + 1}$

use The Quotient Rule to show that the derivative is given by  $\frac{dy}{dx} = \frac{11}{(3x + 1)^2}$

[ 3 marks ]

**Question 4**

Given that  $y = \frac{x^2 + 1}{x^2 + 4}$

use The Quotient Rule to show that the derivative is given by  $\frac{dy}{dx} = \frac{6x}{(x^2 + 4)^2}$

[ 3 marks ]

**Question 5**

Given that  $y = \frac{x^5}{(2x + 1)^3}$

use The Quotient Rule to show that the derivative is given by  $\frac{dy}{dx} = \frac{x^4(4x + 5)}{(2x + 1)^4}$

[ 4 marks ]

**Question 6**

Given that  $y = \frac{x^7}{(3x + 2)^5}$

use The Quotient rule to show that the derivative is given by  $\frac{dy}{dx} = \frac{2x^6(3x + 7)}{(3x + 2)^6}$

[ 5 marks ]

**Question 7**

Given that  $y = \frac{2(x+3)^3}{\sqrt{x}}$

use The Quotient Rule to show that  $\frac{dy}{dx} = \frac{(x+3)^2(5x-3)}{x^{\frac{3}{2}}}$

[ 4 marks ]

**Question 8**

Given that  $y = x^2\sqrt{x+5}$

use The Product Rule to show that  $\frac{dy}{dx} = \frac{5x(x+4)}{2\sqrt{x+5}}$

[ 5 marks ]

**Question 9**

$$f(x) = \frac{2x}{x+5} + \frac{6x}{x^2+7x+10} \quad x > 0$$

(a) Show that

$$f(x) = \frac{2x}{x+2}$$

(b) Hence find  $f'(3)$

[ 5 marks ]

**Question 10**

$$f(x) = \frac{x}{x^2 + 2}$$

Given that  $f(x)$  is increasing on the interval  $[-k, k]$  find the largest possible value of  $k$

**[ 5 marks ]**