

## Lesson 3

### A-Level Pure Mathematics : Year 2 Differentiation III

#### 3.1 Product In, Product Out (PIPO)

When using The Product Rule, the object before differentiation is a product. It's considered elegant to have an object after the differentiation that's also a product. Furthermore, in tackling optimisation problems in which local minima and maxima are sought, (which correspond to where the derivative, the gradient, is zero) having a product equalling zero (rather than a sum) is a desirable situation. In short, initially applying The Product Rule is often only half of a question; manipulating the algebra to derive an answer in the form of a product is the other.

#### 3.2 Example

Show that the derivative of  $y = x^3(2x + 5)^3$  can be expressed as,

$$\frac{dy}{dx} = 3x^2(2x + 5)^2(4x + 5)$$

Teaching Video : <http://www.NumberWonder.co.uk/v9028/3.mp4>



Watch the video and  
then write out the  
solution here



[ 5 marks ]

### 3.3 One For You To Do

Often, in the middle of using The Product Rule, The Chain Rule is required.

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**The Chain Rule for  $y = [f(x)]^n$**

$$\text{If } y = [f(x)]^n \text{ then } \frac{dy}{dx} = n [f(x)]^{n-1} f'(x)$$

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**The Product Rule**

$$\text{If } f = uv \text{ then } f' = uv' + u'v$$

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Try this problem, then check your solution with mine on the following page.

**Try 1** Use the product rule to show that the derivative of,

$$y = (2x + 3)^2 (5x - 1)^3$$

is,

$$\frac{dy}{dx} = (2x + 3)(5x - 1)^2(50x + 41)$$

[ 5 marks ]

**Answer to Try 1**

$$y = (2x + 3)^2 (5x - 1)^3$$

$$\frac{dy}{dx} = (2x + 3)^2 \times 3(5x - 1)^2 \times 5 + 2(2x + 3)^1 \times 2 \times (5x - 1)^3$$

$$\frac{dy}{dx} = (2x + 3)(5x - 1)^2 \{ 15(2x + 3) + 4(5x - 1) \}$$

$$\frac{dy}{dx} = (2x + 3)(5x - 1)^2 \{ 30x + 45 + 20x - 4 \}$$

$$\frac{dy}{dx} = (2x + 3)(5x - 1)^2 \{ 50x + 41 \}$$

**3.3 Exercise**

Marks Available : 35

**Question 1**

Use the product rule to show that the derivative of,

$$y = x^5(x - 1)^2$$

is,

$$\frac{dy}{dx} = x^4(x - 1)(7x - 5)$$

**[ 5 marks ]**

**Question 2**

Use the product rule to show that the derivative of,

$$y = x^7(6x + 5)^3$$

is,

$$\frac{dy}{dx} = 5x^6(6x + 5)^2(12x + 7)$$

[ 5 marks ]

**Question 3**

Use the product rule to show that the derivative of,

$$y = (x^2 - 3)(x + 1)^2$$

is,

$$\frac{dy}{dx} = 2(x^2 - 1)(2x + 3)$$

[ 5 marks ]

**Question 4**

Use the product rule to show that the derivative of,

$$y = (4x + 1)^{\frac{3}{2}}(x^2 + 5)$$

is,

$$\frac{dy}{dx} = 2\sqrt{4x + 1}(7x^2 + x + 15)$$

[ 5 marks ]

**Question 5**

Use the product rule to show that the derivative of,

$$y = (2x - 3)^3(x^2 + 1)^2$$

is,

$$\frac{dy}{dx} = 2(2x - 3)^2(x^2 + 1)(7x^2 - 6x + 3)$$

[ 5 marks ]

**Question 6**

Use the product rule to find the derivative of,

$$y = x^3(5x + 1)^2$$

Write your answer as a product.

[ 5 marks ]

**Question 7**

Find the  $x$  component of the coordinates of the stationary points on the curve

$$y = (x^2 - 1) \sqrt{1 + x}$$

[ 5 marks ]