Lesson 13

A-Level Pure Mathematics : Year 2
Differentiation III

13.1 Later Date Revision

Marks Available : 40

Table of Standard Derivatives

$f(x)$	$f'(x)$	In Formula Book ?
x^n	$n\,x^{n-1}$	No
e^x	e^x	No
$\ln x$	$\frac{1}{x}$	No
$\sin x$	$\cos x$	No
$\cos x$	$-\sin x$	No
$\tan x$	$\sec^2 x$	Yes
$\csc x$	$-\csc x \cot x$	Yes
$\sec x$	$\sec x \tan x$	Yes
$\cot x$	$-\csc^2 x$	Yes
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$	Yes
$\arccos x$	$-\frac{1}{\sqrt{1-x^2}}$	Yes
$\arctan x$	$\frac{1}{1+x^2}$	Yes

Question 1

Show that the derivative with respect to x of;

$$y = \sec x \tan x$$

is

$$\frac{dy}{dx} = \sec x\,(2\sec^2 x - 1)$$

[4 marks]

Question 2

Show that the derivative with respect to x of;

$$y = \csc x \cot x$$

is

$$\frac{dy}{dx} = \csc x \left(1 - 2 \csc^2 x \right)$$

[4 marks]

Question 3

Consider the function;

$$f(x) = \frac{8}{(1 - 3x)^3}$$

Show that;

$$f'(1) = \frac{9}{2}$$

[4 marks]

Question 4

C3 examination question from January 2009

Find the equation of the tangent to the curve

$$x = \cos(2y + \pi) \quad \text{at} \quad \left(0, \frac{\pi}{4}\right)$$

Give your answer in the form $y = ax + b$, where a and b are constants to be found.

[6 marks]

Question 5

The curve

$$y = ln\,(x^2 - 3)$$

crosses the x-axis at A and B.

(i) Find the coordinates of A and B

[3 marks]

(ii) The normals at A and B meet at P
Find the coordinates of P

[5 marks]

Question 6

Show that the derivative of the inverse cotangent function

$$y = \operatorname{arccot} x$$

is

$$\frac{dy}{dx} = -\frac{1}{1 + x^2}$$

The following trigonometry formula will be useful;

$$\cot^2 y + 1 = \csc^2 y$$

[6 marks]

Question 7

The curve

$$y = \frac{2x + 1}{2x - 1}$$

crosses the x-axis at A and the y-axis at B.

Find the point of intersection of the tangents to the curve at A and B.

[8 marks]