

MATHEMATICS PRIZE 1974

The questions are grouped in sections of increasing difficulty (and value) and may be attempted in any order. The writing out of proofs, together with essential working, will carry as much credit as the accuracy of answers.

SECTION A

1. A (convex) polygon has 144 vertices. How many diagonals does it have ?
2. A man set out to cover a certain distance at a certain average speed. He covered half the distance at half the speed; how many times faster must he cover the remaining distance ?
3. A point P is chosen at random on the circle circumscribing a rectangle ABCD. Prove that the value of the expression $PA^2 + PB^2 + PC^2 + PD^2$ does not depend on the position chosen for P.
4. A says : "B did it" B says : "A is lying"
C says : "I did not do it" D says : "A did it"
a) If only one statement is true, who did it ?
b) If only one statement is false, who did it ?
c) What are the other possibilities ?
5. At a certain party, some (but not necessarily all) of the guests shake hands with each other. Prove that the number of guests who shake hands an odd number of times must itself be even.
6. Complete the following cross-number (in base 10 - one digit in each square). The letters a, b, c, d stand for positive integers.

Across : 1. $b-2$ Down : 1. c^2-1 3. a^2-c 2. b^2+d^2 5. $3(a+d)$ 4. $4a+2c$

1	2	/	/
3	7	4	
/	5	8	

SECTION B

7. The lines k, l, m form an equilateral triangle. Identify on a diagram the set of points in the plane which are nearer to k than to l or m.
8. Show that $2n^3 + 2n^2 + 2n + 1$ is never divisible by 3 for any integer n.
9. Horatio has seven girl-friends. Each girl has seven boy-friends. The first girl sends a Valentine to just one of her boy-friends, the second sends one to two, and so on, the seventh girl sending one to all seven of her boy-friends. Find the probabilities
 - a) that Horatio receives seven Valentines
 - b) that he receives only one
 - c) How many times more likely is he to receive six than seven ?
(Leave any powers and factorials in your answer)

TURN OVER

