SHREWSBURY SCHOOL

MATHEMATICS PRIZE, 1962

1. A and B run a mile race, and A wins by 80 yards. When A and C run over the same course, A wins by 20 seconds. B and C run, and B wins by 5 seconds. In what time can A run the mile ?

2. (*a*) Find the factors of 3599.

   (*b*) Is 2477 a prime number ? Find two whole numbers x and y such that $x^2 + y^2 = 2477$.

3. In a system of numeration with "radix" r, the number which is written as $a_3a_2a_1a_0$, where each of a_3, a_2, a_1 and a_0 is less than r, stands for the number $a_3r^3 + a_2r^2 + a_1r + a_0$.

[Thus, in our decimal system (radix 10) 2387 stands for $2\times10^3+3\times10^2+8\times10+7$].

   (*a*) Numbers to be employed in calculations by an electronic computer are expressed in the "binary" system of numeration (radix 2).

   Find the number in our own decimal system which is fed into a computer as 101001.

   (*b*) What is the radix of the system of numeration in which 363 stands for "our" number 507 ?

4. Factorize the following :-

   (i) $x^{16}-16$ (ii) $343a^3+729b^3$ (iii) $32p^2+204pq-81q^2$.

5. If $\dfrac{x}{b+c-a} = \dfrac{y}{c+a-b} = \dfrac{z}{a+b-c}$, shew that

$$(a+b+c)(yz+zx+xy)=(x+y+z)(ax+by+cz).$$

6. Prove that, in any triangle, the sum of the medians lies between the sum of the sides and three-quarters of the sum of the sides.

[P.T.O.

7. Two unequal circles intersect at A and B. PQ is a common tangent to the circles. Prove that AB produced passes through the mid-point of PQ.

The co-ordinates of two points C and D are (1, 1) and (2, 2) respectively, the unit distance on each axis of co-ordinates being one inch.

Give a brief but clear description of the construction of the *two* circles which may be drawn to pass through C and D and touch the x-axis. (A neat sketch should be given, but the accurate construction need not be performed.)

Calculate the distance between the centres of the two circles.

8. The tangents at the points B and C to the circle circumscribed about the triangle ABC meet at the point A′. D is the middle point of BC. Prove that AA′=AD sec A and that the angle CAD is equal to the angle BAA′.