

# SHREWSBURY SCHOOL

## MATHEMATICS PRIZE, 1958

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1. The two factors of 62677 are prime, and differ by less than 50. Find them.

2. Solve the equation

$$\sqrt{x^2+3x+3} - \sqrt{x^2+3x-5} = \sqrt{2x+9} - \sqrt{2x+1}$$

3. A "Halt" sign at a roadside is in the form of a vertical equilateral triangle, with one side horizontal, and is facing due south. Find the angles of its shadow on the ground when the sun is shining from the south-west at an elevation of  $30^\circ$ .

4. A line is said to be divided in "Golden Section" if the ratio of the smaller part to the greater is equal to the ratio of the greater part to the whole.

Calculate the ratio thus defined, and describe a geometrical construction for the division of a line AB in Golden Section.

5. Find the values of  $x$  satisfying the equation

$$a(b-c)x^2 + b(c-a)x + c(a-b) = 0$$

6. ABCD is a square inscribed in a circle. PQRS is a square inscribed in the minor segment AB, with two vertices on AB and two on the circumference of the circle. Prove that  $AB=5PQ$ .

7. Two circles cut at A and B. Describe and justify a construction for a circle which shall touch these two circles in such a way that the line joining the points of contact shall pass through A or B.

8. A set of dominoes, each showing two numbers, can be arranged thus:

- (0, 0)
- (0, 1) (1, 1)
- (0, 2) (1, 2) (2, 2)
- (0, 3) (1, 3) (2, 3) (3, 3)

etcetera, giving 28 dominoes in a set from "double-blank" to "double-six."

In a set from "double-blank" to "double- $p$ ,"

- (i) how many dominoes are there?
- (ii) find the number of times the number  $q$  (less than  $p$ ) appears,
- (iii) the "value" of a domino being the sum of the two numbers appearing on it, find the sum of all the values,
- (iv) show that the average value of a domino is  $p$ .