

# Shrewsbury School

## MATHEMATICS PRIZE, 1950

1. Rewrite this division sum with the correct digits substituted for the  $a$ 's,  $b$ 's and asterisks:—

$$\begin{array}{r}
 \phantom{* a * } * a * \\
 * a * \overline{) * * * * *} \\
 \phantom{* a * } * * * \\
 \phantom{* a * } * * \overline{b b} \\
 \phantom{* a * } * * \overline{b a} \\
 \phantom{* a * } * \overline{a * *} \\
 \phantom{* a * } * * * *
 \end{array}$$

2. Solve the simultaneous equations, giving your answers correct to three significant figures:—

$$\frac{2x+4}{3y+1} - \frac{4-x}{3y+3} = 2\frac{1}{2}$$

$$2x - 3y = 3$$

3. Find a number of four digits such that its digits are reversed when it is multiplied by 4.

4. From a point O within a triangle ABC perpendiculars OX, OY, OZ are drawn to the sides BC, CA, AB respectively. Find where O must be taken so that

$$BX^2 + CX^2 + CY^2 + AY^2 + AZ^2 + BZ^2$$

may be as small as possible.

5. Find five whole numbers,  $a, b, c, d, e$ , of which  $a, b, c, d$ , are consecutive, such that

$$a^3 + b^3 + c^3 + d^3 = e^3$$

**TURN OVER**

6. What is the price of eggs per score if an increase in price of sixpence per dozen will result in  $13\frac{1}{3}$  gross less being purchasable for £99?

7. Four monkeys came upon a pile of nuts. The first divided it into four equal piles, one of which he took for himself, together with the one nut that remained over. The second then divided all the nuts that remained into four equal piles, one of which he took for himself, together with the one nut that remained over. The third and fourth monkeys did likewise. After that there still remained enough nuts for them to be divided into four equal piles with one nut over. Find the smallest number of nuts that the original pile could have contained.

8. Find the law governing these two series of numbers, and state the next term in each series :—

(i) 0, 1, 2, 9, 44, 265, .....

(ii)  $\frac{1}{1}$ ,  $\frac{3}{2}$ ,  $\frac{7}{5}$ ,  $\frac{17}{12}$ ,  $\frac{41}{29}$ , .....

9. Two motor cyclists are racing one another round a circular track  $x$  yards in length. They start together and go in the same direction. The slower one takes  $y$  seconds longer than the faster to complete a lap, and they are together again for the first time after the start in  $k$  minutes. Express the speeds of both men in yards per second in terms of  $x$ ,  $y$ , and  $k$ .

If  $x=240$ ,  $y=3$ ,  $k=1$ , give the numerical answers.

10. A, B, C, D are four points in order on the circumference of a circle. AB, DC meet when produced at O. The line through O parallel to BC meets AD produced at E. A circle with centre E and radius EO is drawn to cut the first circle at F. Prove that EF is a tangent to the first circle.