

# SHREWSBURY SCHOOL

## ARITHMETIC PRIZE, 1943

1. A passenger on a ship 225 yds. long, steaming at 23 m.p.h. observes that waves travelling in the same direction as the ship pass the stern at intervals of 19 secs., and that a wave travels the length of the ship in 15 secs. Find the distance between crests of successive waves.

2. A man borrows £500 on Jan. 1st, 1900 and pays 3% on the amount on loan each year. He repays £80 on Dec. 31st of each year, but misses 1904. On what date is he clear of debt, and how much is his last payment (to the nearest £).

3. Factorise (i)  $96x^2 - 79x - 135$

(ii)  $8x^2 - 6xy - 9y^2 + 10x + 21y - 12$

(iii)  $abc^2 - a^2 - a^2c + b^2c^2$

(iv)  $x^4 + x^2(a^2 + 4b^2) + a^4 + 4b^4$ .

4. In a race of  $x$  yards, A gives B  $y$  yards start and wins by  $t$  secs. But on the same course and at the same speeds B, with  $z$  yards start beats A by  $p$  yards. Find A's time for this course.

5. A damaged tank has a hole in it through which water escapes at a steady rate, emptying it in  $g$  hours. Two pumps working together can fill this tank in  $b$  hours. They work for  $c$  hours when No. 1 pump breaks down. After  $d$  hours the pump is repaired and resumes work delivering twice as much water per hour as before. In  $f$  hours more the tank is full. How long would it take No. 2 pump to fill the damaged tank working alone?

6. Prove that the sum of the medians of a triangle is less than the perimeter but greater than  $\frac{3}{4}$  of the perimeter.

7. PQ and RS are perpendicular chords of a circle. Prove that  $PR^2 + QS^2$  is equal to the square of the diameter of the circle.

8. PQRS is a quadrilateral in which PQ and SR meet at B, and SP and RQ meet at A. The circles APQ, BQR meet at O. Prove that the circle BSP passes through O and find three other points concyclic with O.

9. In a tetrahedron OABC,  $OA = OB = OC = 10''$ ;  $\angle AOB = 40^\circ$ ; and  $AB = BC = CA$ . P and Q are points on OB and OC. Find the least perimeter of the triangle APQ.